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Positive parity low-spin states of even–odd $^{129-133}$Ba isotopes

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Abstract

The positive parity low-spin states of even-odd $^{129-133}$Ba isotopes were studied in this study using the Interacting Boson–Fermion Model (IBFM-1). The single fermion is predicted to be in one of three single-particle orbits: 2d5/2, 2d3/2, and 3s1/2. The Interacting Boson Model (IBM-1) was used to investigate the energy levels, electric quadrupole transition probabilities, and potential energy surface of even-even Barium isotopes (a core for even-odd nuclei).

The measured positive parity low-state energy spectra and predicted energy levels, as well as the B(E2) transition probabilities, are reasonably consistent with the experimental data and previous research for Ba isotopes. The potential energy surface contour plot reveals that all interesting nuclei are deformed and have γ-unstable-like properties.

Keywords: IBM; IBFM; Energy level; Electromagnetic transition; Potential energy surface

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1. Introduction

Traditional descriptions of the nuclear structure of the Barium region have proven difficult to interpret [1]. These nuclei were distinguished by shape changes ranging from spherical to deformed [2]. The IBM-1 has a group structure U(6). This Hamiltonian's three limiting symmetries, U(5), SU(3), and O(6), corresponding to the geometrical shapes, spherical vibrator, symmetric rotor, and γ-unstable rotor, respectively [2]. Barium nuclei have been successfully treated as exhibiting the O(6) symmetry of the Interacting Boson Model (IBM-1) in calculations [3,4]. The IBM has succeeded in reproducing nuclear collective levels in terms of s and d bosons, which are fundamentally the collective s and d pairs of valence nucleons with angular momentum L=0 and 2, respectively [5–8]. The Interacting Boson–Fermion Model (IBFM-1) was proposed by Arima and Iachello [9,10] and describes odd-mass nuclei as systems of fermions coupled to the IBM core via an appropriate Boson-Fermion Interaction.

The IBM and IBFM can be unified into a superalgebra U(6/m), where the dimension of the boson space is 6, and $m_j = \sum_{j_i}(2j_i + 1)$ for the fermion space with angular momentum ($j = j_1, j_2, \ldots$). The even-odd Ba isotopes, together with the even–even Ba isotopes, were studied as an example of a U(6/12) super-symmetry [11–14], in which the odd nucleon could occupy single-particle orbits $j = 1/2, 3/2$, and $5/2$. In recent years, extensive research has been conducted on the structure of the Barium nucleus. The study of the geometry of the Ba isotopes discovered that when the neutron number decreases from N=73 to N=77, the absolute minimum of the potential for the Ba isotopes evolves from spherical to oblate and finally to prolate shapes [15]. The interacting boson model calculations, with or without the inclusion of intruder states in the even $^{129-133}$Ba nuclei, yielded identical energy spectra and absolute B(E2) values up to excitation energy of about 1.5 MeV [16]. The shape/phase transition in Ba nuclei was calculated using the IBM and the systematics of the spectra, as well as the reduced E2 transition probabilities B(E2) [17]. In the considered Ba isotopic chain, the prolate-to-oblate shape/phase transition was shown to occur quite smoothly as a function of neutron number N, with γ-softness playing an essential role. The evolution of the deformation parameter β and of the isotope shifts for a chain of Ba isotopes with the IBM-CM (the CM means configuration mixing) approach has been studied [18]. The total energy surface and the nuclear shape in the isotopic chain $^{172-194}$Pt have been calculated using the interacting boson model, including even-even and even-odd Pt isotopes within the IBM and the IBFM to give a comprehensive view of these isotopes in a rather simple way [19]. The results of the IBFM multilevel calculations for $^{129-133}$Ba isotopes will be presented for energy levels and transition probabilities and will be compared with the corresponding experimental data. The IBM-1 will apply to calculate the low-energy levels according to the arrangement of bands (gr-, γ- and β-) and the B(E2) value for even-even $^{128-132}$Ba isotopes. Then, using the potential energy surface E(N, β, γ), investigate the nuclear structure as described for Ba isotopes.

2. Theory

2.1. The Interacting Boson Model (IBM-1)

According to the Interacting Boson Model (IBM), low-lying collective states in medium and heavy nuclei away from closed shells are controlled solely by excitations of the valence protons and neutrons (i.e., particles outside the major closed shells at 2, 8, 20, 28, 50, 82, and 126), while the closed shell core is inert. In the present case, the valence protons and neutrons are counted from the nearest closed shells, Z=50 and N=82. Furthermore, it is assumed that the coupled particle configurations (identical particles) form pairs of angular momentum 0 and 2. These protons (neutrons) pairs are treated as bosons. Bosons with angular momentum L=0 are denoted by $s_\pi(s_\nu)$ and are called s-bosons, while the bosons with angular momentum L =2 are denoted by $d_\pi(d_\nu)$ and are called d-bosons. The underlying structure of the six-dimensional unitary group U(6) of the model leads to a simple Hamiltonian, capable of describing the three specific types of collective structure with classical geometrical analogs: vibrational U(5), rotational $SU(3)$, and γ-unstable O(6). Hamiltonian H can be explicitly written in terms of boson creation (s†, d†) and annihilation (˜s, ˜d) operators [20] so that,

$$\begin{aligned} \mathrm{H} =&\varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) \\ &+ \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L \left[\left[d^\dagger \times d^\dagger\right]^{(L)} \times \left[\tilde{d}\times\tilde{d}\right]^{(L)}\right]^{(0)} \\ &+ \frac{1}{\sqrt{2}}\upsilon_2\left[\left[d^\dagger \times d^\dagger\right]^{(2)} \times \left[\tilde{d}\times\tilde{s}\right]^{(2)} + \left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d}\times\tilde{d}\right]^{(2)}\right]^{(0)} \\ &+ \frac{1}{2}\upsilon_0\left[\left[d^\dagger \times d^\dagger\right]^{(0)} \times \left[\tilde{s}\times\tilde{s}\right]^{(0)} + \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{d}\times\tilde{d}\right]^{(0)}\right]^{(0)} \\ &+ \frac{1}{2}u_0\left[\left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{s}\times\tilde{s}\right]^{(0)}\right]^{(0)} + u_2\left[\left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d}\times\tilde{s}\right]^{(2)}\right]^{(0)} \end{aligned} \tag{1}$$

It can be written in general form as [20]

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{p}.\hat{p} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \tag{2}$$

Where: $\hat{n}_d = \left(d^\dagger.\tilde{d}\right)$ the total number of d boson operator; $\hat{p} = 1/2\left[\left(\tilde{d}.\tilde{d}\right) - (\tilde{s}.\tilde{s})\right]$ the pairing operator; $\hat{L} = \sqrt{10}\,\left[d^\dagger \times \tilde{d}\right]^1$ the angular momentum operator; $\hat{Q} = \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]^{(2)} + \chi\left[d^\dagger \times \tilde{d}\right]^{(2)}$ the quadrupole operator where χ denotes the quadrupole structure parameter and the values are 0 and $\pm\frac{\sqrt{7}}{2}$ [20,21]; $\hat{T}_r = \left[d^\dagger \times \tilde{d}\right]^{(r)}$ is the octoupole and hexadecapole operator; and $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy.

The phenomenological parameters a_0, a_1, a_2, a_3, and a_4 designated the strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons, respectively. The Hamiltonian operator for O(6) dynamical symmetry is [2,9]:

$$\hat{H} = a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \tag{3}$$

Where the parameters it contain are a_0, a_1 and a_3 only. The Eigen value for this dynamical can be written as [2]:

$$E = A\,(N-\sigma)\,(N+\sigma+4) + B\tau\,(\tau+3) + CL\,(L+1) \tag{4}$$

Where ($A = a_0/4$, $B = a_3/2$, $C = a_{1-}a_3/10$); N is the boson number, $\sigma = N, N-2, \ldots, 0$ or 1.

The O(6) are labeled by the quantum number; $\tau = \sigma, \sigma-1, ..., 0$; $L = 2\lambda, 2\lambda-2, \ldots, \lambda+1$ here λ is non-positive integer defined by $\lambda = \tau - 3\upsilon_\Delta$, $\upsilon_\Delta = 0, 1$; υ_Δ is number of triplet bosons.

σ *is* the irreducible representations (irreps) of O(6), while τ is the irreps of O(5).

2.2. The Interacting Boson–Fermion Model (IBFM-1)

The Interacting Boson-Fermion Model (IBFM) has as its building blocks a set of N-bosons with $L = 0, 2$ and an odd nucleon (either a proton or a neutron). In the odd-A nuclei are described by the coupling of the odd fermionic quasi-particle to a collective boson core. The total Hamiltonian can be written as the sum of three-parts [11,21]:

$$H = H_B + H_F + V_{BF} \tag{5}$$

Where H_B is the IBM Hamiltonian [2,4] for the even-even core, H_F is the Fermion Hamiltonian containing only one-body terms [8,11,21].

$$H_F = \sum_{j\mu} \varepsilon_j a^{\dagger}_{j\mu} \tilde{a}_{j\mu} \tag{6}$$

Where ε_j are the quasi-particle energies and $a^{\dagger}_{jm}\tilde{a}_{jm}$ is the creation (annihilation) operator for the quasi-particle in the eigenstate $|\mathrm{jm}\rangle$; V_{BF} is the boson-fermion interaction that describes the interaction between the odd quasi-nucleon and the even-even core nucleus:

$$\begin{aligned} V_{BF} = &\sum_j A_j \left[\left(d^{\dagger} \times \tilde{d} \right)^{(0)} \times \left(a^{\dagger}_j \times \tilde{a}_j \right)^{(0)} \right]^{(0)}_0 + \sum_{jj'} \Gamma_{jj'} [Q^{(2)} \times \left(a^{\dagger}_j \times \tilde{a}_{j'} \right)^{(2)}]^{(0)}_0 \\ &+ \sum_{jj'j''} \Lambda^{j''}_{jj'} : [\left(d^{\dagger} \times \tilde{a}_j \right)^{(j'')} \times \left(\tilde{d} \times a^{\dagger}_{j'} \right)^{(j'')}]^{(0)}_0 : \end{aligned} \tag{7}$$

V_{BF} is dominated by three terms: The first term is a monopole interaction characterized by the parameter A_0 which plays a minor role in actual calculations:

$$A_j = A_0 \sqrt{2j+1} \tag{8}$$

The most important arises from the quadrupole interaction are the second and third terms [11,12] characterized by Γ_0 and the exchange of the quasi-particle with one of the two fermions forming a boson characterized by Λ_0 and $Q^{(2)}$ is the core boson quadrupole operator, this exchange force is a consequence of the Pauli principle for the quadrupole interaction between protons and neutrons [21]. The remaining parameters in Eq. (8) can be related to the Bardeen-Cooper-Schrieffer (BCS) theory occupation probabilities u_j and ν_j of the single-particle orbits by [11,12,16]:

$$\Gamma_{jj'} = \sqrt{5}\Gamma_0 \left(u_j u_{j'} - \nu_j \nu_{j'} \right) Q_{jj'} \tag{9}$$

$$\Lambda^{j''}_{jj'} = -\sqrt{5}\Lambda_0 \frac{\left[\left(u_{j'}\nu_{j''} + \nu_{j'}u_{j''} \right) Q_{j'j''}\beta_{j''j} + \left(u_{j'}\nu_{j''} + \nu_{j'}u_{j''} \right) Q_{j''j}\beta_{j'j''} \right]}{\sqrt{2j''+1}} \tag{10}$$

Where $Q_{jj'}$ and $\beta_{jj'}$ are the matrix elements of the quadrupole operator on a single-particle basis and the structure coefficients of the d boson deduced from microscopic considerations, respectively [12,22],

$$\left. \begin{aligned} Q_{jj'} &= \langle j \| Y^{(2)} \| j' \rangle \\ \beta_{jj'} &= Q_{jj'}(u_j \nu_{j'} + \nu_j u_{j'}) \end{aligned} \right\} \tag{11}$$

The BCS calculation is used to generate the occupation probabilities u_j and ν_j. The quasi-particle energy ε_j of each single-particle orbital can be obtained by solving the gap equations [12,15]:

Table 1
Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, excepted N.

Isotopes	N	EPS	PAIR	ELL	Q.Q	OCT	HEXA
^{128}Ba	8	0.0	0.048	0.016	0.0	0. 033	0.0
^{130}Ba	7	0.0	0.072	0.034	0.0	0.036	0.0
^{132}Ba	6	0.0	0.106	0.056	0.0	0.042	0.0

$$\left.\begin{array}{l} \varepsilon_j = \sqrt{\left[\left(E_j - \lambda\right)^2 + \Delta^2\right]} \\ \nu_j^2 = 1/2\left[1 - \left(E_j - \lambda\right)/\varepsilon_j\right] \\ u_j^2 = 1 - \nu_j^2 \end{array}\right\} \tag{12}$$

Where E_j is the single-particle energy calculated from the relations in [13], λ is the Fermi level energy and Δ is the pairing gap energy, which was chosen to be 12 $A^{-1/2}$ MeV [23]. That leaves the strengths A_0, Γ_0, and Λ_0 as free parameters, which are varied to give the best fit to the excitation energies.

3. Results and discussion

3.1. Even–even Ba isotopes (core)

This section presented the calculated results of the low-lying states of the even-A nuclei, whose proton number is equal to 56, with neutron numbers ranging from 72 to 76. The results include energy levels, the B(E2) values, and Potential Energy Surface.

3.1.1. Energy levels

The $^{128--132}$Ba nuclei have been described using the IBM-1 Hamiltonian (Eq. (2)). The computer code PHINT, written by Scholten [24], is used to calculate energy levels. The isotopic chains of Barium with a number of bosons ranging from 5 to 7 are used in the IBM-1 framework. Table 1 shows the coefficient values that were used in this study. Fig. 1 depicts the calculated ground (gr-), β_1- and γ_1-bands, as well as experimental data of energy levels. The IBM calculations (energies, spin, and parity) agree well with the experimental results [25]. However, it deviates from the experimental data's high spin (energies). Levels with '()' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally. The γ_2-band and the β_2-band are calculated in this work and are given in Table 2 and Table 3, respectively. These tables show a comparison between the IBM-1 calculated energy levels and the experimental data. From this comparison, we can see a good agreement between experimental data and the IBM-1 calculations. Levels with '*' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

3.1.2. B(E2) values

A successful nuclear model must provide a good description of the nucleus's energy spectrum as well as its electromagnetic properties. In IBM-1, the electromagnetic transitions operator has the general form [20,21,26].

$$T_m^l = \alpha_2 \delta_{l2} \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \beta_l \left[d^\dagger \times \tilde{d}\right]_m^l + \gamma_0 \delta_{l0} \delta_{m0} [s^\dagger \times \tilde{s}]_0^0 \tag{13}$$

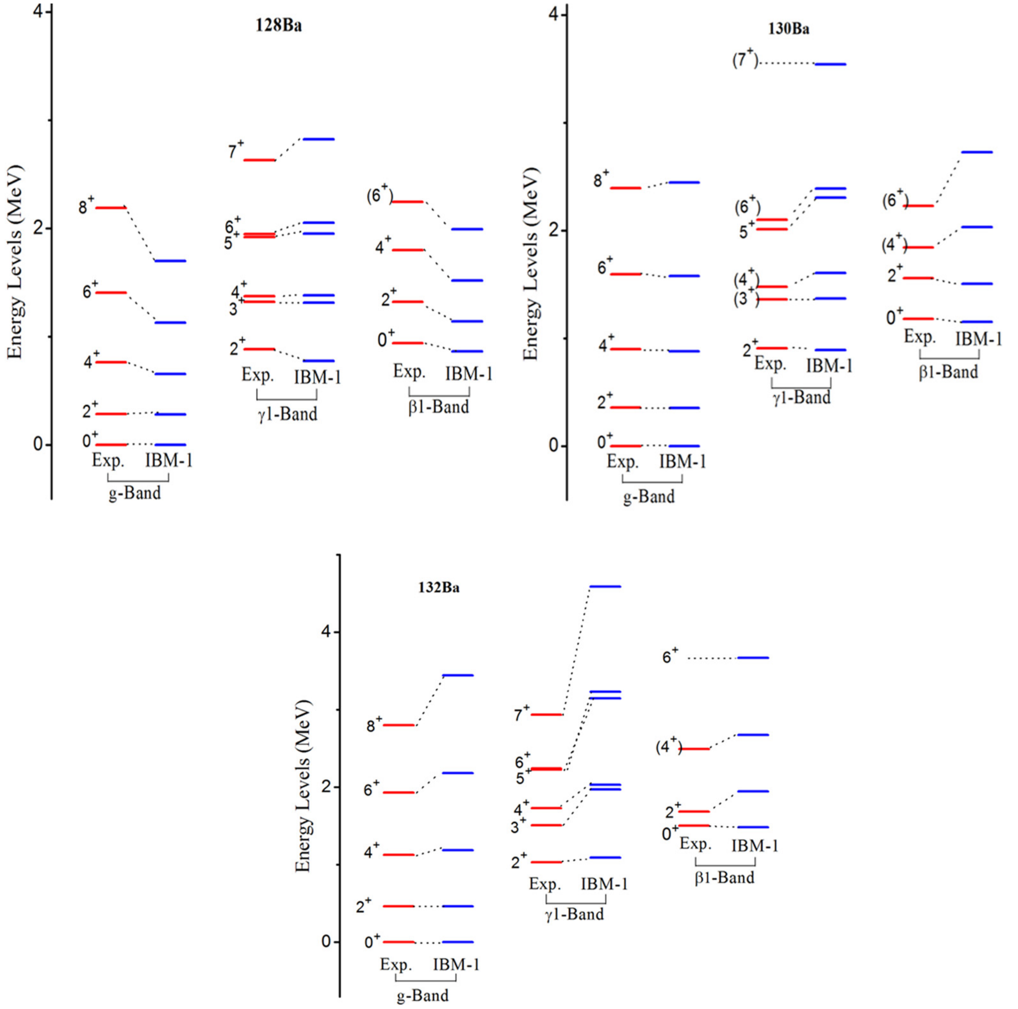

Fig. 1. Comparison the IBM-1 calculations with the available experimental data [25]for $^{128-132}$Ba nuclei.

Table 2
γ_2-bands for Ba isotopes (in MeV). The experimental data are taken from [25].

J^{π}	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{128}Ba		^{130}Ba		^{132}Ba	
2^+	1.8150	2.0390*	2.0460	1.8829	2.5760	1.9981
3^+	2.2470	2.2034*	2.7600	2.0791*	3.4580	3.4238*
4^+	2.1400	1.8337	2.500	2.3179*	3.0800	3.0687*
5^+	2.9190	2.4254*	3.5700	——-	——-	——-

The first term can be presented only in the case of $l = 2$ transitions, while the last term can be presented only in the case of $l = 0$ transitions. This is assured by Kronecker delta (δ) accompanying them. In the special cases of electric monopole, quadrupole and hexadecapole transitions,

Table 3
β_2-bands for Ba isotopes (in MeV). The experimental data are taken from [25].

J^{π}	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{128}Ba		^{130}Ba		^{132}Ba	
0^+	1.4850	1.7100	1.6200	—-	1.8900	1.6600
2^+	2.2590	2.3472*	2.3700	—-	2.9820	2.0464*
4^+	2.1790	1.9077	2.7520	2.7840*	3.5140	3.4949*
6^+	2.8170	2.7211*	3.5940	—	4.7180	—–

Table 4
Effective charge used to reproduce B(E2)values for $^{128--132}$Ba nuclei.

Isotopes	N	B(E2;$2_1^+ \rightarrow 0_1^+$) w.u. [25]	B(E2;$2_1^+ \rightarrow 0_1^+$) e^2b^2	e_B
^{128}Ba	7	72	0.275	0.119
^{130}Ba	6	57.9	0.226	0.121
^{132}Ba	5	43.4	0.173	0.120

the specific form of the transition operator is respectively, γ_0, α_2 and β_l ($l = 0, 1, 2, 3, 4$) which are parameters specifying the various terms in the corresponding operators. Then the electric quadrupole transition is:

$$\begin{aligned} T_m^{E2} &= \alpha_2 \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \beta_2 \left[d^\dagger \times \tilde{d}\right]_m^2 \\ &= \alpha_2 \left(\left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \chi \left[d^\dagger \times \tilde{d}\right]_m^2 \right) \\ &= e_B \hat{Q} \end{aligned} \tag{14}$$

Where α_2 and β_2 are two parameters, and ($\beta_2 = \chi\alpha_2, \alpha_2 = e_B$ (effective charge of boson)) B(E2) values are defined in terms of reduced matrix elements by [20]:

$$B\left((E2 : L_i \rightarrow L_f\right) = \frac{1}{2L_i + 1} \left| \left\langle L_f \left\| T^{(E2)} \right\| L_i \right\rangle \right|^2 \tag{15}$$

Where: $\left| \left\langle L_f \left\| T^{(E2)} \right\| L_i \right\rangle \right|$ is the matrix element of (E2) the electric quadrupole transition. The values of effective charge (e_B) were estimated to reproduce the experimental B(E2;$2_1^+ \rightarrow 0_1^+$) from the selection rules ($\sigma = 0$ and $\tau = \pm 1$) [10,20] and it is tabulated in Table 4. The computer code BEFM, written by Scholten [24], is used to calculate B(E2) values. Table 4 shows the comparison of calculated B(E2) values with experimental data [25] for all nuclei under consideration. Table 5 shows that the B(E2; $2_1^+ \rightarrow 0_1^+$) values are increase as neutron number increases toward the middle of the shell in Ba nuclei, in general, there are a good agreement between the calculated B(E2)and the experimentally data.

3.1.3. B(M1) values and E2/M1 mixing ratios

To investigate M1 transitions in the IBM-1 framework, it has therefore been necessary to introduce second-order terms [6]. Then the magnetic dipole operator was written as [2,4,12]:

$$T(M1) = (g_B + AN)\hat{L} + B[\hat{T}(E2) \times \hat{L}] + C\hat{n}_d\hat{L} \tag{16}$$

Table 5
The IBM-1 and the experimental values of B(E2)for $^{128--132}$Ba nuclei (in $e^2.b^2$).

$J_i \to J_f$	IBM-1	EXP. [25]	IBM-1	EXP. [25]
	^{1128}Ba		^{130}Ba	
$2_1^+ \to 0_1^+$	0.2719	0.27	0.2255	0.22
$2_2^+ \to 2_1^+$	0.3682	0.2034	0.3012	0.3002
$4_1^+ \to 2_1^+$	0.3682	0.4100	0.3012	0.30
$4_2^+ \to 2_2^+$	0.2225	0.2400	0.1662	0.1700
$4_2^+ \to 4_1^+$	0.1888	—-	0.1511	—-
$5_1^+ \to 4_2^+$	0.0922	—-	0.0711	—-
$5_1^+ \to 6_1^+$	0.0491	—-	0.0379	—-
$6_1^+ \to 4_1^+$	0.3965	0.3900	0.3172	0.3700
$6_2^+ \to 4_2^+$	0.2633	0.2800	0.2033	0.2100
$6_2^+ \to 6_1^+$	0.1229	—-	0.0949	—-
$8_1^+ \to 6_1^+$	0.3862	0.3800	0.2981	0.2800
$J_i \to J_f$	^{1128}Ba			
$2_1^+ \to 0_1^+$	0.1728	0.17		
$0_2^+ \to 2_2^+$	0.2263	0.2233		
$2_2^+ \to 2_1^+$	0.2263	—-		
$4_1^+ \to 2_1^+$	0.1207	0.2300		
$4_2^+ \to 2_2^+$	0.1097	—-		
$4_2^+ \to 4_1^+$	0.1646	—-		
$5_1^+ \to 4_2^+$	0.02596	—-		
$5_1^+ \to 6_1^+$	0.2304	0.2271		
$6_1^+ \to 4_1^+$	0.1392	0.2200		

Where g_B (atomic number (Z)/mass number (A)) is the effective boson g-factor, N is the number of bosons, $\hat{L}$ is the angular momentum operator, $\hat{T}$(E2) is matrix elements of the E2 operator, $\hat{n}_{\mathrm{d}}$ is d-boson number operator, and the g-factor of the states defined as [2]:

$$g_{\mathrm{L}} = \mu_{\mathrm{L}}/\mathrm{L} \tag{17}$$

Where μ_{L} is the magnetic moments and can be defined as:

$$\mu_{\mathrm{L}} = \sqrt{\frac{4\pi}{3}} \frac{L}{\sqrt{[L(L+1)(2L+1)]}} \langle L \| T(M1) \| L \rangle \tag{18}$$

❖ The magnetic dipole transitions can be calculated for **O(6)** dynamical symmetry. The matrix element of the $\hat{n}_d$ operator can be written as [9]:

$$\left\langle [N], \sigma = N, \tau, \nu, L \left| \hat{n}_d \right| [N], \sigma = N-2, \tau, \nu, L \right\rangle = -\sqrt{N} \sqrt{\left[\frac{N(N+3) - \tau(\tau+3)}{2N(N+1)}\right]} \sqrt{\left[\frac{(N-1)(N-2) - \tau(\tau+3)}{2N(N+1)}\right]} \tag{19}$$

and

$$g_{2_1^+} = g_B + AN + \sqrt{\frac{4\pi}{3}} \frac{4 + N(N-1)}{2(N+1)} C \tag{20}$$

The Ml matrix element which yields from eq. (15) can be written as [20,24]:

Table 6
The coefficients of T(M1) of Ba isotopes used in the present work. All parameters are given in (μ_N), except N.

Isotopes	N	g_B	A	C	B
^{128}Ba	8	0.4375	−0.0530	−0.00003	−0.0263
^{130}Ba	7	0.4307	−0.0115	0.00001	−0.033
^{132}Ba	6	0.4242	−0.0140	−0.000003	−0.0385

$$\langle \grave{\varphi} L_f \| T(M1) \| \varphi L_i \rangle = -Bf(L_i L_f) \langle \grave{\varphi} L_f \| T(E2) \| \varphi L_i \rangle + C[L_i(L_i+1)(2L_i+1)]^{1/2} \times \langle \grave{\varphi} L_f | \hat{n}_d | \varphi L_i \rangle \delta_{L_i L_f} \quad (21)$$

Where $\grave{\varphi}$ and φ denote additional quantum numbers. In the first term the spin factor $f(L_i L_f)$, given separately [7] for the cases $L \to L \pm 1$ and $L \to L$, written as:

$$f(L_i L_f) = \begin{bmatrix} \frac{1}{40}(L_i + L_f + 3)(L_f - L_i + 2) \\ \times (L_i - L_f + 2)(L_i + L_f - 1) \end{bmatrix}^{1/2} \quad (22)$$

The second term of eq. (22) only contributes to transitions between states of the same spin since the corresponding operator in eq. (21) is diagonal in $\hat{L}$. For $L \pm 1 \to L$ transitions, Eq. (21) leads to a particularly simple expression for the reduced E2/M1 mixing ratio, namely [20,24]:

$$\Delta(E2/M1) = \langle \grave{\varphi} L_f \| T(E2) \| \varphi L_i \rangle / \langle \grave{\varphi} L_f \| T(M1) \| \varphi L_i \rangle = -1/Bf(L_i L_f) \quad (23)$$

The reduced mixing ratio $\Delta(E2/M1)$ is related to the quantity normally measured by [26]:

$$\delta(E2/M1) = 0.835[E_\gamma/(1\ \text{MeV})]\Delta(E2/M1) \quad (24)$$

Where: E_γ is in MeV and $\Delta(E2/M1)$ is in eb/μ_N. D. P. Grechukhin [27] has already derived the spin dependence $f(L_i L_f)$ of eq. (22) in the framework of the geometrical model in an analogous way, by expressing the relevant part of the M1 operator in terms of the quadrupole coordinates of the nuclear surface. Similarly, to calculate the B(M1) values by using the computer codes BEFM, we must specify the values of the parameters g_B, A, C, and B. All these parameters are tabulated in Table 6.

The comparison of the calculated B(M1) values with the experimental data [25] are given in Table 7 for all isotopes under study. The B(M1) of ^{128}Ba for $2_2^+ \to 2_1^+$ and $4_2^+ \to 4_1^+$ transitions are 0.0002 and 0.0003 which agreement with the experimental data 0.0001 and 0.0002, respectively. For ^{132}Ba, the $2_2^+ \to 2_1^+$ transition is 0.0002, which is in disagreement with the experimental value 0.0017.

The Interacting Boson Model has been applied to calculate the E2/M1 multipole mixing ratios over a wide range of nuclei. The multipole mixing ratios; In the transition nucleus, even-parity states of even-even nuclei are ascribed to the collective quadrupole motion of the nucleus as a whole the δ(E2/M1) multipole mixing ratios of the electromagnetic transitions between the energy states of $^{128-132}$Ba nuclei were calculated by using equations (23) and (24) and given in Table 8. In general, it can be seen from the table that calculated results are not in better agreement with the experimental data [25] for some Ba isotopes under study. That the mixing ratio found for ^{128}Ba the $2_2^+ \to 2_1^+, 3_1^+ \to 2_1^+, 3_1^+ \to 4_1^+, 4_2^+ \to 4_1^+$ transitions are 18.0903, 19.1427, 18.6761 and 13.8412, respectively. These values are in agreement with the experimental value of $13(^{+16}_{-4})$, $4(^{+12}_{-1})$, $3.7(^{+25}_{-12})$ and $-14(^{+8}_{-16})$, respectively. For ^{132}Ba, the $3_1^+ \to 2_1^+, 3_1^+ \to 2_2^+, 3_1^+ \to 4_1^+$ which agreement with the experimental values of 20(8), −21(10) and 0.9000, respectively. For

Table 7
B(M1) values for Barium nuclei (in μ_N^2).

$J_i \to J_f$	^{128}Ba		^{130}Ba		^{132}Ba	
	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
$2_2^+ \to 2_1^+$	0.0002	0.0001	0.0002	——	0.0002	0.0017
$3_1^+ \to 2_2^+$	0.0002	——	0.0002	——	0.0002	——
$3_1^+ \to 4_1^+$	0.0001	——	0.0001	——	0.0001	——
$4_2^+ \to 4_1^+$	0.0003	0.0002	0.0004	——	0.0004	——
$5_1^+ \to 4_2^+$	0.0002	——	0.0003	——	0.0002	——
$5_1^+ \to 6_1^+$	0.0002	——	0.0002	——	0.0002	——
$6_2^+ \to 6_1^+$	0.0005	——	0.0006	——	0.0005	——
$7_1^+ \to 6_2^+$	0.0001	——	0.0001	——	0.0001	——
$7_1^+ \to 8_1^+$	0.0003	——	0.0003	——	0.0002	——

Table 8
The IBM-1 and the experimental values of δ(E2/M1) multipole mixing ratios for $^{126-130}$Ba nuclei.

$J_i \to J_f$	^{128}Ba				^{130}Ba			
	E_γ (MeV)		δ (E2/M1)		E_γ (MeV)		δ (E2/M1)	
	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
$2_2^+ \to 2_1^+$	0.4950	0.600	18.0903	$13(^{+16}_{-4})$	0.5400	0.5500	17.5094	—-
$3_1^+ \to 2_1^+$	1.1040	1.0380	19.1427	$4(^{+12}_{-1})$	1.0140	1.0030	28.5112	20(8)
$3_1^+ \to 2_2^+$	0.6090	0.4370	19.1353	—-	0.4880	0.4590	10.8300	−21(10)
$3_1^+ \to 4_1^+$	0.7280	0.5590	18.6761	$3.7(^{+25}_{-12})$	0.7200	0.5757	11.6843	0.9000
$4_2^+ \to 4_1^+$	0.6600	0.6090	13.8412	$-14(^{+8}_{-16})$	0.8840	0.5350	14.0121	—-
$5_1^+ \to 4_2^+$	0.7400	0.5590	13.2670	—-	0.9120	0.4190	13.1458	—-
$6_2^+ \to 6_1^+$	0.8250	0.5320	10.8001	—-	0.9000	0.5070	9.4511	—-

$J_i \to J_f$	^{132}Ba			
	E_γ (MeV)		δ (E2/M1)	
	IBM-1	EXP.	IBM-1	EXP.
$2_2^+ \to 2_1^+$	0.6300	0.5670	17.7075	$14(^{+3}_{-2})$
$3_1^+ \to 2_1^+$	0.8820	0.4790	21.1370	4.0 (12)
$3_1^+ \to 4_1^+$	0.7840	0.3830	14.8120	6 (11)
$4_2^+ \to 4_1^+$	0.8400	0.6017	17.7075	—-
$5_1^+ \to 4_2^+$	1.1200	—	26.8296	—-
$5_1^+ \to 6_1^+$	0.9660	—	11.5258	—-
$6_2^+ \to 6_1^+$	1.0500	—-	9.9964	—-

^{132}Ba the $2_2^+ \to 2_1^+, 3_1^+ \to 2_1^+, 3_1^+ \to 4_1^+$ transitions are 17.7075, 21.1370, and 14.812 that values are in agreement with the experimental values of $14(^{+3}_{-2})$, 4.0 (12), and 6 (11) respectively. Furthermore, two experimental values have a negative sign and the theoretical calculations have a positive sign because the absolute sign of the mixing ratios is a matter of convention, a change in the sign from one nucleus to another can be indicative of a change in nuclear shape since the mixing ratio is proportional to the quadrupole operator.

3.1.4. Potential energy surface (E(N, β, γ))

The potential energy surface gives a final shape to the nucleus that corresponds to the function of Hamiltonian [28], as in the following equation [20]:

$$E(N,\beta,\gamma) = \langle N,\beta,\gamma \mid H \mid N,\beta,\gamma\rangle / \langle N,\beta,\gamma \mid N,\beta,\gamma\rangle \tag{25}$$

The expectation value of the IBM-1 Hamiltonian with the coherent state ($|$N, β, $\gamma_{-}\rangle$) is used to create the IBM energy surface [20,29]. The state is a product of boson creation operators ($b_c^\dagger$), with

$$\Big| N,\beta,\gamma\rangle = 1/\sqrt{N!}\left(b_c^\dagger\right)^N |0\rangle \tag{26}$$

$$b_c^\dagger = \left(1+\beta^2\right)^{-1/2}\left\{s^\dagger + \beta\left[\cos\gamma\left(d_0^\dagger\right) + \sqrt{1/2}\sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right]\right\}$$

The energy surface, as a function of β and γ, has been given by [2]:

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{\left(1+\beta^2\right)^2}\left(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4\right) \tag{27}$$

Where the αi's are related to the coefficients C_L, ν_2, ν_0, u_2 and u_0 of eq. (1). And β is a measure of the total deformation of nucleus, where $\beta = 0$ the shape is spherical, and is distorted when $\beta \neq 0$, and γ is the amount of deviation from the focus symmetry and correlates with the nucleus, if $\gamma = 0$ the shape is prolate, and if $\gamma = 60$ the shape becomes oblate. The contour plots in the γ–β plane, see Fig. 2, resulting from E(N, β, γ) are shown for $^{128-132}$Ba isotopes. For most of the considered Ba nuclei the mapped IBM energy surfaces are γ-unstable shapes. γ-unstable shape is associated with intermediate values $0 < \gamma < \pi/3$. The γ-unstable deformation helps to understand the prolate-to-oblate shape transition that occurs in the considered Ba isotopes. The Ba nuclei considered here do not display any rapid structural change but remain γ-soft. The absence of any prolate to oblate transition in $^{128-132}$Ba is consistent with the parameter-free algebraic predictions of Ref. [30], in which it is clear that prolate to oblate transitions can be expected in the Hf-Pt region.

3.2. Even–odd Ba isotopes

This section presented the calculated results of the low-lying states of the even-A nuclei, whose proton number is equal to 56, with neutron numbers ranging from 73 to 77. The results include energy levels and the B(E2)values.

3.2.1. Energy levels

In recent years, many positive parity states of the even–odd nuclei, such as even–odd Barium isotopes have been found experimentally. Below the shell closure N=82 one finds the positive parity levels 2d5/2, 2d3/2, 3s1/2, which form the fermionic U(12) algebra. This is already a severe approximation, since in the standard Nilsson diagrams the 1g7/2 level lies above the 2d5/2 level, but the 2d5/2 level is preferred to close the fermionic algebra. The negative parity level 1h11/2 is already present at the same energy as 2d3/2, but it is ignored because of its parity. However, this is again a severe approximation, since the 1h11/2 level is known to play a fundamental role in the gradual development of nuclear deformation, according to Refs. [31–33]. The basic algebraic structure associated with the IBFM Hamiltonian of the nucleus, whose last, unpaired nucleon occupies single-particle orbits with j=1/2, 3/2 and 5/2, is the direct product $U^B(6) \times U^F(12)$, where

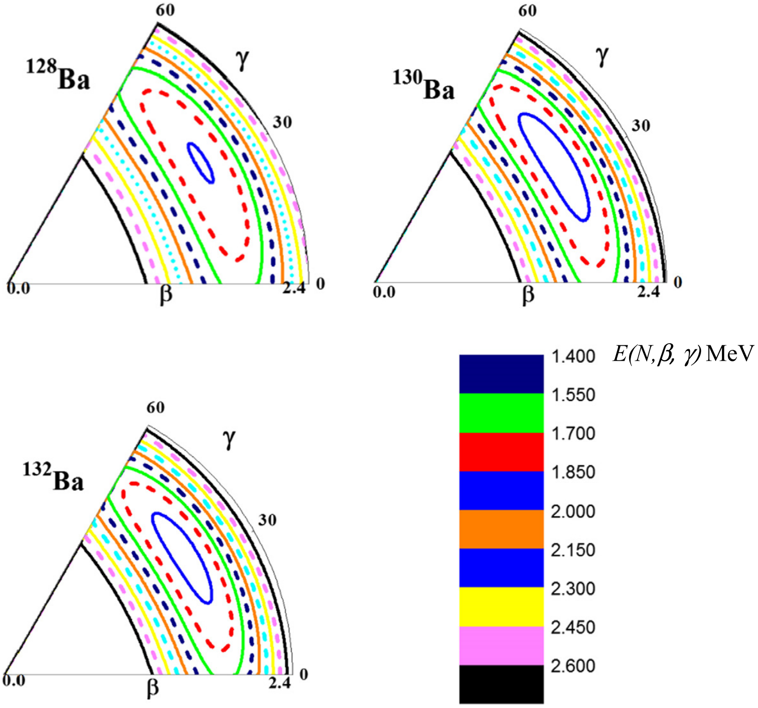

Fig. 2. The potential energy surface in γ–β plane for $^{128--132}$Ba nuclei. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

$U^B(6)$ is the boson group describing the collective properties of the even–even core and $U^F(12)$ is the fermion group associated with the single-particle degree of freedom. In the framework of the IBFM-1, we performed the BCS calculation, which provided the quasi-particle energies (ε_j) and shell occupancies (υ_j^2). The adopted ε_j, υ_j^2 are presented in Table 9. From Table 9, we can see that the occupation probabilities increase as the neutron number increases. The IBFM Hamiltonian (eq. (5)) was diagonalized by means of the computer program ODDA [34] in which the IBFM parameters are identified as A0=BEM, Γ0=BFQ and Λ0=BFE. In the present work, the parameters for the $^{128--132}$Ba core were derived. The IBFM parameters used in ODDA are given in Table 10 for all nuclei under study.

The IBFM-1 calculation and the experimental data of low-lying states were plotted in Figs. 3 for the even–odd $^{129--133}$Ba isotopes. In these figures, the IBFM calculations (energies, spin and parity) are in good agreement with the experimental data [25]. The levels with '()' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

From Fig. 3 for $^{129--133}$Ba isotopes, it can be noticed that for a great number of bosons (N), there are more energy levels converging with each other because of increasing number of energy levels with boson number (N) increment. Furthermore, it is confirmed the levels the $1/2_3$ with energy 0.547 MeV, for ^{129}Ba isotope. In addition, the predicted levels of new energy 0.651 MeV with spin (parity) $9/2_2$. The levels $3/2_2$, $5/2_2$, $7/2_2$ and $9/2_2$ have energies 0.340, 0.364, 0.789 and 0.856 MeV, respectively for ^{131}Ba isotope. As well as, the predicted levels of new energy

Table 9
Adopted values for the parameters used for IBFM-1 calculations. The parameter ε_j is given in MeV.

Parameters	^{129}Ba			^{131}Ba		
	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$
ε_j	1.34	1.06	1.05	1.21	1.05	1.048
ν_j^2	0.34	0.44	0.49	0.36	0.45	0.49
	^{133}Ba					
ε_j	1.36	1.064	1.048			
ν_j^2	0.37	0.48	0.52			

Table 10
Adopted parameters which is used for IBFM calculations. All parameters are given in MeV.

A	^{129}Ba	^{131}Ba	^{133}Ba
BFE	0.11	0.205	0.009
BFQ	0.01	0.01	0.00
BFM	−0.19	−0.27	0.01

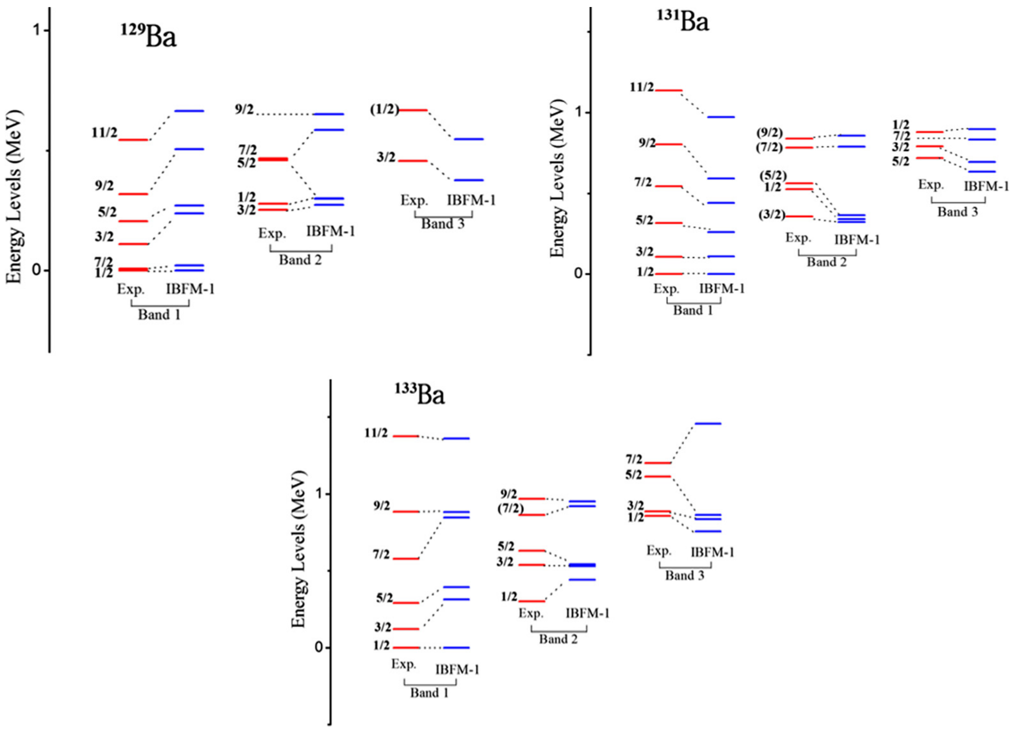

Fig. 3. Comparison the IBM-1 calculations with the available experimental data [25] for $^{129-133}$Ba nuclei.

0.832 MeV with spin (parity) $7/2_3$. The level $7/2_2$ has energy 0.919 MeV for ^{133}Ba isotope, respectively.

Table 11
The coefficients of T^{E2} for $^{129-133}$Ba nuclei used in the present work.

Isotopes	e_B(eb)	e_F(eb)
^{129}Ba	0.119	0.3856
^{131}Ba	0.121	−0.3579
^{133}Ba	0.120	−0.3520

3.2.2. B(E2)values

The calculation of electromagnetic transitions gives a good test of the nuclear model wave functions. In this section we discuss the calculation of E2 transition strengths and compare the results with the available experimental data. In general, the electromagnetic transition operators can be written as the sum of two terms, the first of which acts only on the boson part of the wave function and the second only on the fermion part [10]. In the IBFM the E2 operator is:

$$T_\mu (E2) = e_B \hat{Q}_{B,\mu} + e_F \sum_{jj'} \hat{Q}_{jj'} [a_j^\dagger \times \tilde{a}_{j'}]_\mu^{(2)} \tag{28}$$

Where $\hat{Q}_{B,\mu}$B and $\hat{Q}_{jj'}$ are the boson and fermion quadrupole operators and e_B and e_F are the effective boson and fermion charges. The values of effective charge (e_B) was calculated previously (Table 4), and the values of effective charge (e_F) are estimated from the selection rules $\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3 = 0$ and $\Delta(\tau_1 + \tau_2) = \pm 1$ transitions, which are allowed for $e_B(\alpha_2) = e_F(f_2)$. $\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3 \neq 0$, $\Delta(\tau_1 + \tau_2) = \pm 1$ transitions, which are only allowed for $e_B(\alpha_2) \neq e_F(f_2)$, At $e_B(\alpha_2) \neq e_F(f_2)$, the effective charge ($e_F$) can be reproduced from the experimental B(E2; $J_i \to J_f$) and can be written as

$$B(E2; (N,1,0), (\tau_1,\tau_2)_i, L_i, J_i \to (N+1,0,0), (\tau_1,\tau_2)_f, L_f, J_f = (\alpha_2 - f_2)^2 \frac{2N(N+3)}{5(N+1)(N+2)} \tag{29}$$

The values of effective charge (e_F) are tabulated in Table 11. The comparison of the calculation of B(E2) values with experimental data [25] is given in Table 12 for all Ba isotopes. It is in good agreement with the experimental data of B(E2) values.

3.2.3. B(M1) values and E2/M1 mixing ratios

For even-odd nuclei and in contrast to even-even nuclei ($NF = 0$) where M1 transitions are largely retarded, magnetic dipole occur in odd-even nuclei ($NF = 1$), with considerable strength. The M1 operator can be written as [35]:

$$T_\mu^{(M1)} = T_{B,\mu}^{(M1)} + T_{F,\mu}^{(M1)} \tag{30}$$

$$T_{B,\mu}^{(M1)} = \beta_1 \left[d^\dagger \times \tilde{d}\right]_\mu^1, T_{F,\mu}^{(M1)} = f_1 [a_j^\dagger \times \tilde{a}_{j'}]_\mu^{(1)} \tag{31}$$

Introducing the operators $B^{(1)}$ and $A^{(1)}$, can be rewrite eq. (28) as [35]:

$$T_\mu^{(M1)} = \beta_1 B_\mu^{(1)} - \frac{t_1}{\sqrt{2}} A_\mu^{(1)} \tag{32}$$

Where $f_1 = t_1/\sqrt{2}$, $\beta_1 = t_1$ the M1 operator can be written as [35]:

Table 12
B(E2) values for even-odd Barium nuclei [25] (in $e^2.b^2$).

$J_i \to J_f$	IBFM-1	EXP.	IBFM-1	EXP.
	^{129}Ba		^{131}Ba	
$5/2_1 \to 1/2_1$	0.0561	——	0.0665	——
$3/2_1 \to 1/2_1$	0.0348	——	0.0722	0.0946
$1/2_2 \to 3/2_1$	0.0368	——	0.0389	——
$3/2_2 \to 3/2_1$	0.0164	——	0.0157	——
$3/2_2 \to 1/2_1$	0.0344	——	0.0067	——
$3/2_2 \to 1/2_2$	0.4446	——	0.4624	——
$3/2_1 \to 5/2_1$	0.0002	——	0.0731	——
$5/2_2 \to 3/2_2$	0.6234	——	0.6459	——
$7/2_1 \to 5/2_1$	0.0825	——	0.1025	——
$7/2_2 \to 5/2_2$	0.3559	——	0.3686	——
$9/2_1 \to 7/2_1$	0.2707	——	0.2806	——
$J_i \to J_f$	^{133}Ba			
$5/2_1 \to 1/2_1$	0.0540	0.0103		
$5/2_2 \to 1/2_2$	0.0637	0.0397		
$3/2_1 \to 1/2_2$	0.0375	——		
$3/2_1 \to 1/2_1$	0.0349	<0.0714		
$3/2_2 \to 1/2_1$	0.0077	——		
$5/2_1 \to 3/2_1$	0.4471	0.1412		
$5/2_2 \to 3/2_2$	0.0001	——		
$7/2_1 \to 5/2_1$	0.6246	——		
$7/2_2 \to 5/2_2$	0.0979	0.0469		
$9/2_1 \to 7/2_1$	0.3565	——		

$$T_{\mu}^{(M1)} = \beta_1 G_{\mu}^{(1)} \tag{33}$$

Since G(1) is proportional to the total angular momentum operator, the only non-zero matrix elements of the operator (eq.33) are diagonal and all M1 transitions are forbidden. Furthermore, the equality $t_1 = \beta_1$ is a poor approximation compared to actual microscopic calculations. We shall therefore consider, in this case, the general form (eq.31), with $t_1 \neq \beta_1$. The operator (eq.32) has the selection rules $\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3$ =0, and $\Delta(\tau_1 + \tau_2)$= 0, ±1 [35].

The most general form of the M1 transition operator can be written as [35,36]:

$$T_{B,\mu}^{(M1)} = \beta_1 \left[d^{\dagger} \times \tilde{d}\right]_{\mu}^{1} + \sum_{jj'} f_{jj'}^{(1)} [a_j^{\dagger} \times \tilde{a}_{j'}]_{\mu}^{(1)} \tag{34}$$

Realistic values of the coefficients $f_{jj'}^{(1)}$ can be obtained by relating them to the matrix elements of the single particle operators, $\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3$ =0

$$f_{jj'}^{(1)} = -\frac{f_1}{\sqrt{3}} \left\langle j \left\| g_l \overrightarrow{l} + g_s \overrightarrow{s} \right\| j' \right\rangle \tag{35}$$

Where $\vec{l}$ is the orbital angular momentum of the odd nucleon, $\vec{s}$ is the spin angular momentum, g_l and g_s are the orbital and spin g-factors and f_1 is the overall strength. In the calculations of B(M1) values for the odd-mass $^{129-133}$Ba isotopes, the collective g-factors and are taken from a study of magnetic properties in the even-even Ba nuclei. The single-particle values of the g-factors are, $g_l = 0$ and $g_s = -3.82$ for a neutron and $g_l = 1$ and $g_s = 5.58$ for a proton [37].

Table 13
B(M1) values for even-odd Barium nuclei (in μ_N^2).

$J_i \to J_f$	^{129}Ba		^{131}Ba		^{133}Ba	
	IBFM-1	EXP.	IBFM-1	EXP.	IBFM-1	EXP.
$3/2_1 \to 1/2_1$	0.0187	——	0.0181	0.0483	0.0176	>0.0411
$1/2_2 \to 3/2_1$	0.0670	——	0.0652	——	0.0636	——
$3/2_2 \to 1/2_1$	0.0001	——	0.0001	——	0.0001	——
$1/2_2 \to 3/2_2$	0.0005	——	0.0005	——	0.0005	0.0020
$5/2_1 \to 3/2_2$	0.0063	——	0.0063	——	0.0063	——
$5/2_2 \to 3/2_2$	0.0498	——	0.0496	——	0.0495	——
$7/2_1 \to 5/2_1$	0.0002	——	0.0002	——	0.0002	——
$7/2_1 \to 5/2_2$	0.0714	——	0.0692	——	0.0673	——
$7/2_2 \to 5/2_2$	0.0006	——	0.0006	——	0.0006	——
$9/2_1 \to 7/2_1$	0.0487	——	0.0471	——	0.0458	——
$11/2_1 \to 9/2_1$	0.0007	——	0.0007	——	0.0006	——

Table 14
The IBFM-1 and the experimental values of δ(E2/M1) multipole mixing ratios for $^{129-133}$Ba nuclei.

$J_i \to J_f$	^{129}Ba				^{131}Ba			
	E_γ (MeV)		δ (E2/M1)		E_γ (MeV)		δ (E2/M1)	
	IBFM-1	EXP.	IBFM-1	EXP.	IBFM-1	EXP.	IBMF-1	EXP.
$3/2_1 \to 1/2_1$	0.1046	0.1101	0.1191	——	0.1101	0.1081	0.1806	0.1270 (14)
$1/2_2 \to 3/2_1$	0.2020	0.1860	0.1250	——	0.2122	0.2570	0.1368	——
$3/2_2 \to 1/2_1$	0.2372	0.2530	3.6773	——	0.3048	0.2852	2.0846	——
$1/2_2 \to 3/2_2$	0.0694	0.0248	1.7293	——	0.0820	0. 1620	2.0836	——
$5/2_2 \to 3/2_2$	0.1204	0.2033	0.3557	<0.6	0.0202	0.2780	0.0609	——
$7/2_1 \to 5/2_1$	0.0959	0.0310	1.6270	——	0.1809	0.2260	3.4208	——
$9/2_1 \to 7/2_1$	0.7109	0.2554	0.5028	——	0.1510	0.2611	0.3077	——

$J_i \to J_f$	^{133}Ba			
	E_γ (MeV)		δ (E2/M1)	
	IBFM-1	EXP.	IBFM-1	EXP.
$3/2_1 \to 1/2_1$	0.0126	0.0120	0.0154	≤0.013
$1/2_2 \to 3/2_1$	0.1369	0.5270	0.0877	——
$3/2_2 \to 1/2_1$	0.1620	0.3021	1.1877	——
$3/2_2 \to 1/2_2$	0.0125	0.1360	0.1178	——
$5/2_2 \to 3/2_2$	0.2959	0.3281	0.0110	——
$7/2_1 \to 5/2_1$	0.1724	0.2860	0.8048	——
$9/2_1 \to 7/2_1$	0.2964	0.3050	0.6905	——

The calculated reduced probability for M1 transitions and the experimental data [25] are given in Table 13.

The calculated mixing ratio and the experimental data [25] are given in Table 14 for the $^{129-133}$Ba isotopes. It can be seen that the calculated results are in fair agreement with the available experimental data. The mixing ratio found for ^{129}Ba the $5/2_2 \to 3/2_2$ transition is 0.3557 that value is in agreement with the experimental value of < 0.6. For ^{131}Ba nucleus, the $3/2_1 \to 1/2_1$ transition is 0.1806. This value is in agreement with the experimental values of 0.1270 (14). For ^{133}Ba nucleus, the $3/2_1 \to 1/2_1$ transition is 0.0154. This value is in agreement with the experimental values of ≤ 0.013.

4. Conclusions

Based on the IBFM-1, we analyzed the even-odd $^{129--133}$Ba isotopes in this paper. A single fermion is coupled to the even-even core of $^{128--132}$Ba isotopes to describe the nuclei. An IBM-1 analysis was used to obtain the boson core parameters, and the main results for the energy levels and electric transition probabilities agree well with the experimental data. Furthermore, our calculations support certain energy levels. The potential energy surface for even-even Ba isotopes reveals that all nuclei are deformed and have O(6) dynamical symmetry. The results show that the energy spectra of the even-odd Ba isotope can be quite well reproduced. The calculated B(E2) and B(M1) values are consistent with the experimental data. The E2/M1 mixing ratios for even-even $^{128--132}$Ba and even-odd $^{129--133}$Ba isotopes with neutron numbers ranging from 72 to 77 are calculated. Furthermore, the calculated results of the 133Ba nucleus have been compared to previous studies, and they are better than of Ref. [38].

We conclude that general characteristics of Ba isotopes are accounted for -unstable shapes are supported in this region. This study's findings confirm that this technique should be expanded to investigate the nuclear structure of other nuclei near the A $\sim$ 130 mass.

CRediT authorship contribution statement

Saher M. Mutsher: Data curation, Writing – original draft. **Fadhil I. Sharrad:** Conceptualization, Methodology, Software, Supervision, Writing – review & editing. **Emad A. Salman:** Supervision.

Declaration of competing interest

The authors note that they have no established conflicting financial interests or personal relationships that the work reported in this paper may have appeared to affect.

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