PAPER • OPEN ACCESS

Phenomenological Description of $^{130,\ 131}$Ba Isotopes

View the article online for updates and enhancements.

Phenomenological Description of $^{130,\ 131}$Ba Isotopes

Saher M. Mutsher, Emad A. Salman and Fadhil I. Sharrad

[1]College of Medicine, University of Thi-Qar, Thi-Qar, Iraq
[2]Department of Physics, College of Science, University of Thi –Qar
[3]Department of Physics, College of Science, University of Kerbala, Iraq.
4College of Health and Medical Technology, Al-Ayen University, Al Nasiriya, Thi Qar, Iraq.
fadhil.altaie@gmail.com

Abstract: The phenomenological description of Barium nuclei which have proton number (56) and neutron numbers 74 and 75 have been calculated by Interacting Boson – Fermion Model. Yrast and excited bad, probability of electric transition B(E2) and potential energy surface have been estimated. The input parameters which are used in the present study the best approximation that has been carried out so far. These calculations show that a good agreement with those of experimental data for Barium nuclei.

Keywords: IBFMl, Ba nuclei, Energy levels, B(E2) Values.

1. Introduction

In the atomic structure, more models used to clarify the atomic structure one of these models is Interacting Boson Model (IBM). It is recommended by Arima and Iachello to portray the atomic structure for even-even cores [1, 2]. It has been extensively applied to the structure of yrast and energized state in even proton and even neutron of center core and has critical accomplishment. In the IBM, the even-even center is believed to be a collection of conveying s and d bosons with saucy power (L)= 0 and 2, exclusively. This model is connected with a natural social occasion structure, which allows the introduction of limiting adjusts called U(5), SU(3) and O(6) [1, 3]. The partner boson model addresses an enormous development forward in our understanding of nuclear structure. It offers a fundamental Hamiltonian, prepared for depicting total nuclear properties over a wide extent of centers, and is set up on rather wide numerical get-together theoretical systems, which have in like manner found progressing application to issues in atomic, sub-nuclear, and high-essentialness material science [4]. The utilization of this model to deformed centers is starting at now a subject of critical interest and discussion. The conveying boson model (IBM-1) [5] and its growth to the odd-A centers, the interfacing boson-fermion model (IBFM-1) [6], have wind up being prepared to give a powerful portrayal of extensively changing classes of centers masterminded away from shut shell structures.

The even proton and even neutron of Barium cores, Ba (Z=56), are one of the most noteworthy centers which depicted by shape changes among roundabout and distorted. Various exploratory and speculative assessments on the structure of imperativeness level and electromagnetic change properties of the even-even phenomenal earth isotopes had been investigated [7-17].

The purpose of the current work is to do a Phenomenological examination of the even-even and even-odd Ba isotopes inside the IBM and the IBFM to give a comprehensive viewpoint on these isotopes in rather direct way. The outcomes of the IBFM amazed means $^{130,\ 131}$Ba isotopes will present for essentialness levels and changes probabilities and will differentiate and the contrasting the preliminary data. In like manner, the IBM-1 will apply to calculate the low-essentialness levels as demonstrated by strategy of gatherings (gr-, γ-and β-) and the B(E2) regard for even-even $^{130,\ 131}$Ba isotopes by then examination of the wave work structure for Ba isotopes.

2. Theory

The nuclear model (interacting boson fermion model) its structure hinders a lot of numbers of boson which is distributed in the angular orbitals L=0 and 2. Moreover, the odd nucleon proton or neutron, and M-fermions involving single-molecule circles with rakish second $j_i = j_1,\ j_2,\ j_3,\ …..$

The segments of the fermion precise second are the m-dimensional space of the gathering $U(m)$ with $m_j = \sum_{j_i}(2j_i + 1)$. The fermions creation $a_i^\dagger$ and destruction $\tilde{a}_i$ administrators are for the single-molecule notwithstanding. The boson creation $b_i^{\dagger}$ and obliteration $\tilde{b}_i$ are administrators for the aggregate degrees of opportunity. The fermion administrators fulfill hostile to compensation relations [18-20]:

$$\{\tilde{a}_i, a_j^\dagger\} = \delta_{ij}, \ \{a_i^\dagger, a_j^\dagger\} = \{\tilde{a}_i, \tilde{a}_j\} = 0 \tag{1}$$

The linear finding of fermion creation $a_i^\dagger$ and destruction $\tilde{a}_j$

The Hamiltonian of Interacting Boson Fermion Model had been depend on the algebraic structure, that aim of concurrent probability of dynamical limit for even-odd nuclei. In case of the single-j, the m values are $m = 2j + 1$, in general, a chain of algebras is:

$$U(2j + 1) \supset SU(2j + 1) \supset SP(2j + 1) \supset SU(2) \supset O(2) \tag{2}$$

In present work, the yrast and excited state for ^{130,131}Ba nuclei have been calculated by IBFM. Spin, parity and energy of levels, the reduced probability of electric transition are calculated. These calculations compared with those of experimental data.

In the IBFM odd-A cores are portrayed as far as a blended arrangement of communicating bosons and fermions, the idea of dynamical balances must be summed up. Under the limitation, that both the boson and fermion states have great rakish force, the separate gathering chains ought to contain the revolution bunch O(3) for boson and SU(2) for fermion as subgroup [21,22].

$$\left.\begin{array}{l} U^B(6) \supset …………… O^B(3) \\ U^F(m) \supset ……… SU^F(2) \end{array}\right\} \tag{3}$$

On the off chance that one of subgroups of UB(6) is isomorphic to one of the subgroups of UF(m), the boson and fermion bunch chains can be joined into a typical boson-fermion bunch chain. At the point when the Hamiltonian is written as far as Casimir invariants of the joined boson-fermion bunch chain, dynamical boson-fermion evenness emerges. The odd-A cores are depicted by the coupling of the odd fermionic semi molecule to an aggregate boson center. The complete Hamiltonian comprises of three sections and is given by the accompanying condition [23, 24]:

$$H = H_B + H_F + V_{BF} \tag{4}$$

Which contains one-body terms only and given by

$$H_F = \sum_{j\mu} \varepsilon_j\, a_{j\mu}^\dagger \tilde{a}_{j\mu} \tag{5}$$

The ε_j are the energies of quasi-particle and $a_{jm}^\dagger \tilde{a}_{jm}$ is the creation (annihilation) operator for the quasi-particle in the Eigen function $|jm\rangle$.

And V_{BF} is describes of the interaction quasi-nucleon with the core (even-even) nucleus that's mean V_{BF} represented the interaction between fermion and the core nucleus [25-28]:

$$V_{BF} = \sum_j A_j \left[(d^\dagger \times \tilde{d})^{(0)} \times (a_j^\dagger \times \tilde{a}_j)^{(0)} \right]_0^{(0)} + \sum_{jj'} \Gamma_{jj'} [Q^{(2)} \times (a_j^\dagger \times \tilde{a}_{j'})^{(2)}]_0^{(0)} \quad (6)$$

$$+ \sum_{jj'j''} \Lambda_{jj'}^{j''} : [(d^\dagger \times \tilde{a}_j)^{(j'')} \times (\tilde{d} \times a_{j'}^\dagger)^{(j'')}]_0^{(0)} :$$

where $Q^{(2)}$ is the quadrupole operator for core nucleus.

The A_j, $\Gamma_{jj'}$ and $\Lambda_{jj'}^{j''}$ are parameters and defined in the following equations:

$$\left.\begin{aligned}
A_j &= A_0 \sqrt{2j+1} \\
\Gamma_{jj'} &= \sqrt{5}\, \Gamma_0 (u_j u_{j'} - v_j v_{j'}) Q_{jj'} \\
\Lambda_{jj'}^{j''} &= -\sqrt{5}\Lambda_0 \frac{[(u_j v_{j''} + v_j u_{j''})Q_{j'j''}\beta_{j''j} + (u_j v_{j''} + v_j u_{j''})Q_{j''j}\beta_{jj''}]}{\sqrt{2j''+1}}
\end{aligned}\right\} \quad (7)$$

The A0 is the monopole communication;

Γ0 is the quadrupole communication;

Λ0 is the trading of a semi molecule with one of the two fermions framing a boson.

The dynamical boson-fermion evenness related with as far as ability and the single fermion (odd nucleon) involving single-molecule circles with turn j = 1/2, 3/2, 5/2. For this situation, the fermion space is disintegrated into a pseudo-orbital part with K = 0, 2 and a pseudo-turn part with s = ½ [29].

3. Calculations and Discussion

3-1 Energy states

In Ba isotope has 56 protons and 74-75 neutron numbers. Along these lines, it has one fermion out the center and as per the association boson-fermion model (IBFM) code. Our figuring's have been assessed by IBFM-1 and it is structure didn't qualification among neutron and proton fermion. We are attempted to keep the base of the free boundaries number in Hamiltonian for the count of the excitation energies in Ba cores. The Hamiltonian unequivocal articulation which is utilized to figuring the vitality levels are given by [30]. H=HB+HF+VBF

The boundaries of even-even and even-odd cores that are given the best fitting between the counts and those of exploratory information introduced in the tables 1 and 2. These boundaries are given in MeV unit, excepted N without unit. The vitality conditions of the gr-□ and □-groups for even-even Ba core and the vitality levels of the 1, 2 and 3 groups for even-odd Ba core. The code PHINT is utilized to compute the vitality levels for even-even core. The ODDA code used to compute the vitality levels for even-odd core [31].

Table 1. The parameters of IBM and IBFM have been used in the present calculations. These parameters are in the MeV unit, excepted N without unit.

IBFM			IBM			
BFE	BFQ	BFM	N	PAIR	ELL	OCT
-0.270	0.010	0.205	7	0.073	0.035	0.035

Table 2. The parameters of IBFM have been used in the present calculations. The parameter ε_j is in the MeV unit, excepted v^2_j without unit.

Parameters	^{131}Ba		
	3p$_{1/2}$	*2f$_{5/2}$*	*3p$_{1/2}$*
ε_j	1.21	1.05	1.048
v^2_j	0.36	0.45	0.49

From figure 1, it is noticed that the calculated gr-, γ- and β-bands for even-even nucleus and the calculated 1, 2 and 3-bands for even-odd nucleus with the experimental data [32]. in this figure, the calculated energy states compared with the experimental data and are in good agreement between them for all nuclei. State with "( )" in gr-, γ- and β-bands for even-even nucleus and 1, 2 and 3-bands for even-odd nucleus correspond to cases for which the energy, spin and/or parity of the corresponding states are not well confirm experimentally.
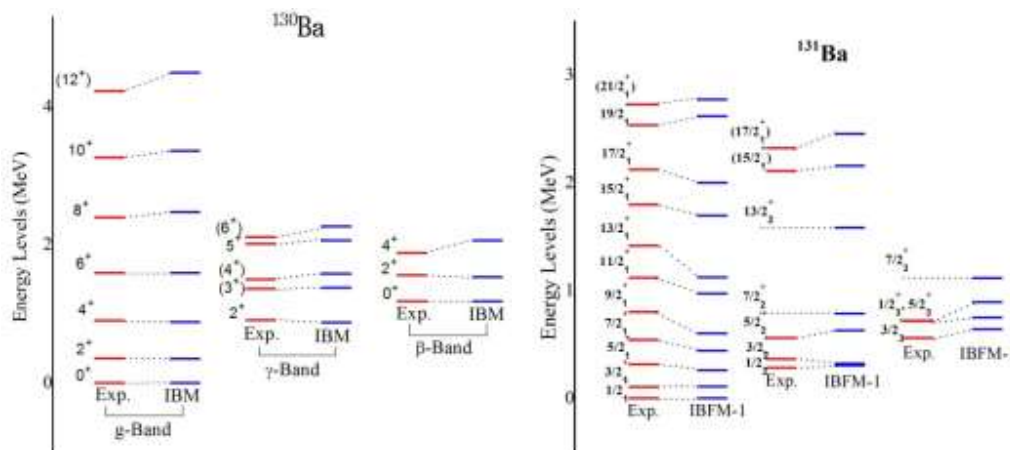
Figure 1: (Color online) the calculated IBFM and IBM compared with the experimental data [32] for ^{130,131}Ba nuclei.

3-2 Reduce Probability Electric Transition *B(E2)Value*

The second property of nuclear structure is electric reduced probability transition. This property will be explained in details with empirical values. Furthermore, it is give a good test of nuclear structure and the wave function of the nuclear model. The electric transition operators can be express as sum of two parts, the first part is for the boson of the eigenstate and second part only on the fermion eigenstate [18,19], the fermion can be moved in *j =1/2, 3/2*, and *5/2* sub- orbital. The (e_B) values represented on the effective charge and calculated form experimental data and it tableted in the table (3). The selections rules which used in the our estimated for the effective charge (e_F) values for fermion are $\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3 = 0$ and $\Delta(\tau_1 +\tau_2)= \pm1$ transitions. It is for $\Delta\sigma1 = \Delta\sigma_2 = \Delta\sigma_3 \neq 0$, $\Delta(\tau_1 +\tau_2)= \pm1$ transitions, $e_B(\alpha_2) = e_F(f_2)$ allowed, and thus expected to be weaker, which are   for $e_B \neq e_F$. At $e_B \neq e_F$ allowed only, the effective charge (e_F) can be reproduced from the experimental   B(E2; $J_i \rightarrow J_f$) and it express as [18].

$$B(E2; (N, 1, 0), (\tau_1, \tau_2)_i, L_i, J_i \rightarrow (N + 1, 0, 0), (\tau_1, \tau_2)_f, L_f, J_f)$$

$$= (\alpha_2 - f_2)^2 \frac{2N(N+3)}{5(N+1)(N+2)} \tag{8}$$

(e_F) represented on the effective charge for fermion and it is tableted in table (3). The B(E2) results are calculated and compared with experimental value and tableted in table (4)

Table 3. The parameters which used in the B(E2) values calculations for ^{130,131}Ba nuclei. These parameters are in eb unit.

Isotope	N	α_2
^{130}Ba	7	0.1213
	e_B(eb)	E_F(eb)
^{131}Ba	0.121	-0.340

Table 4. The B(E2) (in $e^2 b^2$) values which reproduced by IBFM-1, IBM codes and experimental [32].

$J_i \rightarrow J_f$				^{130}Ba	^{131}Ba
	IBM-1	**EXP**.	$J_i \rightarrow J_f$	**IBFM**	**EXP**
$2_1^+ \rightarrow 0_1^+$	0.2266	0.2266	$3/2_1^- \rightarrow 1/2_1^-$	0.0672	0.0672
$2_2^+ \rightarrow 0_1^+$	0.0007	0.0005	$3/2_2^- \rightarrow 1/2_1^-$	0.1702	--
$2_2^+ \rightarrow 2_1^+$	0.3002	0.3200	$3/2_2^- \rightarrow 3/2_1^-$	0.0892	--
$4_1^+ \rightarrow 2_1^+$	0.3027	0.3087	$5/2_1^- \rightarrow 1/2_1^-$	0.0117	--
$6_1^+ \rightarrow 4_1^+$	0.3188	0.3365	$5/2_2^- \rightarrow 1/2_1^-$	0.1828	--
$8_1^+ \rightarrow 6_1^+$	0.2996	0.3404	$5/2_1^- \rightarrow 3/2_1^-$	0.0021	--
$10_1^+ \rightarrow 8_1^+$	0.2547	0.2387	$5/2_2^- \rightarrow 3/2_1^-$	0.0292	--
$12_1^+ \rightarrow 10_1^+$	0.1883	0.1252	$7/2_1^- \rightarrow 3/2_1^-$	0.2330	--

3.3. Potential Energy Surface (PES)

The interfacing boson model was at first written as far as creation and obliteration boson administrators; its mathematical translation fit as a fiddle factors is typically done by presenting the inborn sound state, it is communicated as a boson condensate [25]:

$$|N, \beta, \gamma\rangle = 1/\sqrt{N!} \left(b_c^\dagger \right)^N |0\rangle \tag{9}$$

The $|0\rangle$ represent on the boson vacuum, and

$$b_c^\dagger = (1 + \beta^2)^{-1/2} \left\{ s^\dagger + \beta \left[\cos\gamma \left(d_0^\dagger \right) + \sqrt{1/2} \, \sin\gamma \left(d_2^\dagger + d_{-2}^\dagger \right) \right] \right\} \tag{10}$$

The valence bosons outside the doubly-shut shell are the boson number (N).
The β boundary is identified with the hub disfigurement of the core, while γ measures the deviation from pivotal evenness which decides the mathematical state of the core. There are $\beta \geq 0$

and $0 \leq \gamma \leq \pi/3$ in this study. The calculation of the expectation value of the Hamiltonian (1) on the intrinsic boson condensate state give the expressed of potential energy surface and it can be expressed as (8):

$$E(N,\beta,\gamma) = \langle N,\beta,\gamma|H|N,\beta,\gamma\rangle / \langle N,\beta,\gamma|N,\beta,\gamma\rangle$$

$$= \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}\left(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4\right) \qquad (11)$$

In the U(5), SU(3), and O(6) dynamical limits which correspond to the large N_b, $\beta_{min} = 0, \sqrt{2}$, and 1, respectively. Figure 2 show that the calculated potential energy surfaces for the even-even Ba nucleus with N=74. From this figure, the even-even Ba isotope is deformed and has γ-unstable characters.
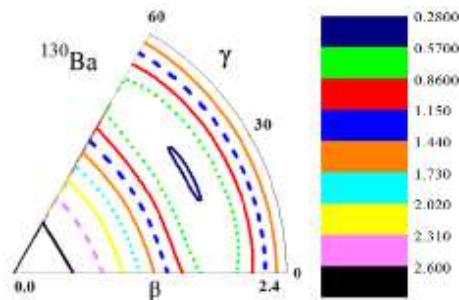
Figure 2. PES for even ^{130}Ba isotope

4. Conclusion

The vitality states and electric decreased change probabilities B(E2) estimations of the gr-, γ-and β-groups for even-even core have been assessed utilizing cooperating boson model. Moreover, 1, 2 and 3-groups for even-odd core have been determined utilizing Interacting Boson-Fermion Model. The potential vitality surface has been determined for even-even core. The cores under investigation are having medium mass. The correlation between the estimations and test information show that the worthy between them. The B(E2) values have been determined and contrasted and test information and it is understanding between them. The computation of potential vitality surface shows that the even-odd core has misshaping shape.

Acknowledgements

References

[1] F. I. Sharrad ,I. Hossain , I. M.Ahmed, H. Y. Abdullah, S. T. Ahmad and A. S. Ahmed
Braz J Phys **45,** 340 (2015).
[2] F. I. Sharrad, H. Y. Abdullah, N. AL-Dahan , N. M. Umran, A. A. Okhunov and H.
Abu-Kassim, Chinese Physics C **37,** 034101 (2013).
[3] I. Hossain, I. M.Ahmed, F. I. Sharrad, H. Y. Abdullah, A. D. Salman and N. AL-Dahan,
Chiang Mai J. Sci.**42** 996(2015).

[4] A Shelley, I Hossain, Fadhil I Sharrad, Hewa Y Abdullah and M A Saeed Prob. Atom. Sci.& Tech. 64 38 (2015).

[5] I. Hossain, H. H. Kassim, F. I. Sharrad and A. S. Ahmed, ScienceAsia **42,** 22(2016).

[6] M. A. Al-Jubbori , H. H. Kassim, F. I. Sharrad and I. Hossain, Nucl. Phys. A **955,** 101(2016).

[7] H. H. Khudher, A. K. Hasan and F. I. Sharrad , Ukrainian Journal of Physics **62,** 152(2017).

[8] M. O. Waheed and F. I. Sharrad, Ukrainian Journal of Physics **62,** 757(2017).

[9] R. F. Casten and D. D. Warner, Rev. Mod. Phys. **60**, 389 (1988).

[10] I. M. Ahmed, G. N. Flaiyh, H. H. Kassim, H. Y. Abdullah, I. Hossain and F. I. Sharrad, Eur. Phys. J. Plus **132**, 84 (2017).

[11] H. H. Kassim and F. I. Sharrad, Nucle. Phys. A **933**, 1(2015).

[12] M. A. Al-Jubbori, F. Sh. Radhi, A. A. Ibrahim, S. A. Abdullah Albakri, H. H. Kassim and F. I. Sharrad, Nuclear Physics A **971,** 35 (2018).

[13] O. Scholten, Computer code PHINT, KVI; Groningen, Holland, (1980).

[14] M. A. Al-Jubbori , H. H. Kassim, F. I. Sharrad , A. Attarzadeh and I. Hossain, Nuclear Physics A **970,** 438 (2018).

[15] A. Okhunov, F. I. Sharrad, A. A. Al-Sammarea and M. U. Khandaker, Chinese Physics C **39**, 084101 (2015).

[16] H. H. Kassim, A. A. Mohammed-Ali, M. Abed Al-Jubbori, F. I. Sharrad, A.S. Ahmed and I. Hossain, J.Natn.Sci.Foundation Sri Lanka **46**, 3 (2018).

[17] M. Abed Al-Jubbori, K. A. Al-Mtiuty, K. I. Saeed and F. I. Sharrad, Chinese Physics C **41**, 084103 (2017).

[18] I. Hossain, F. I. Sharrad, M. A.Saeed , H. Y. Abdullah and S. A. Mansour, Maejo Int. J. Sci. Technol. **10,** 95(2016).

[19] H. H. Kassim and F. I. Sharrad, International Journal of Modern Physics E 23, (2014) 1450070.

[20]. Fadhil I. Sharrad, H.Y. Abdullah, N. Al-Dahan, A. A. Mohammed-Ali, A. A. Okhunov and H. Abu Kassim, Romanian Journal of Physics, 57, (2012) 1346-1355

[21] M A Al-Jubbori, H H Kassim, A A Abd-Aljbar, H Y Abdullah, I Hossain, I M Ahmed and Fadhil I Sharrad, Indian Journal of Physics, 94(2020) 379-390

[22]Wisam N. Hussain, I. Hossain and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012027

[23] Mariam O. Waheed and Fadhil I. Sharrad, , Journal of Physics: Conf. Series, 1279 (2019) 012077

[24] Mahdi A. Mahdi, Fahmi Sh. Radhi, Huda H. Kassim, Mushtaq AbedAl- Jubbori and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012021

[25] Kahtan A. Hussain, Musa K. Mohsin and Fadhil I. Sharrad, , Iranian Journal of Science and Technology, Transactions A: Science,43(2017) 1273–1285

[26] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fahmi Sh. Radhi, Amin Attarzadeh, I. Hossain, Imad M. Ahmed, and Fadhil I. Sharrad, Physics of Atomic Nuclei, 82 (2019) 201–211

[27]Imad Mamdouh Ahmed, Hewa Y. Abdullah, Mudhaffer Mustafa Ameen, Huda H. Kassim and Fadhil I. Sharrad, Physics of Atomic Nuclei, 81 (2018) 695–702

[28]Wisam N. Hussain and Fadhil I. Sharrad, Journal of Physics: Conference Series, 1032 (2018) 012046

[29] Imad Mamdouh Ahmed, Mushtaq Abed Al-Jubbori, Huda H. Kassim, Hewa Y. Abdullah and Fadhil I. Sharrad, Nuclear Physics A, 977 (2018)34-48

[30] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fadhil I. Sharrad and I. Hossain, International Journal of Modern Physic E, 27 (2018) 1850035.

[31]Huda H. Kassim, Amir A. Mohammed-Ali, Fadhil I. Sharrad, I. Hossain and Khalid S. Jassim, Iranian Journal of Science and Technology, Transactions A: Science, 42 (2018) 993-999.

[32] http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp