Barkas correction of particle and antiparticle

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Abstract

The stopping power for a charged particle penetrating through matter differs from the an anti – particle. This difference is called Barkas correction has been studied theoretically as a function of velocity and projectile – target combination. In this paper, the behavior of stopping power and Barkas correction of protons in Aluminum (Al) and Gold (Au) has been studied. Moreover, in this research, a theoretical study was made about the effect of Bark's correction on the density function C(χ) and the way of how it changes, particularly, at low energy and through the Barkas correction, we can distinguish between the particle and anti-particle.

تصحيح باركز للجسيم وضديدة

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الخلاصة:-

إن قدرة إيقاف الجسيم المشحون الذي يخترق المادة يختلف عن قدرة الإيقاف بالنسبة لضديد الجسيم. إن هذا الفرق يدعى تصحيح باركز الذي قد تم دراسته نظريا كدالة لسرعة الجسيم الساقط وتركيب القذيفة – الهدف. في هذا البحث تم دراسة سلوك قدرة الإيقاف وتصحيح باركز للبروتونات في أهداف من الألمنيوم (Al) والذهب (Au). بالإضافة إلى ذلك تم في هذا البحث إجراء دراسة نظرية حول تأثير باركز على دالة الكثافة والطريقة التي يتغير بها وبالأخص عند الطاقة الواطئة والذي من خلاله يمكن التمييز بين الجسيم وضديدة.

1. Introduction:

When a charged particle traverses matter, it will lose energy due to interaction with the target atoms. The energy loss of the projectile per unit distance in the target material is

called the stopping power of the material $(-dE/dx)$. It depends on the charge and velocity of the projectile and, of course, the target material[1].

The stopping power of ions in matter have been considered theoretically since the early days of atomic physics starting with Bohr, Thomson and Rutherford. The interest was first motivated by the necessity to get a good theoretical understanding of the slowing down process in order to extract information above the nature of the studied atomic particles. Furthermore, the analysis of penetration phenomena offered a testing ground for the theoretical treatments being developed, starting with classical methods and subsequently turning to quantum mechanical methods and finally the computer simulation codes which still remain a best tool in the iterative dialogue between theory and experiment[2].

Accurate stopping power data in variety of materials and energies ranges are of practical importance in a number of contemporary experiments used extensively in materials science, such as ion implantation and ion – beam analysis, which require accurate knowledge of stopping power and ranges values[2].

The theory of energy loss of fast charged particles in matter is based on the calculations by Bethe, who derived the stopping power in the first Born approximation. Hence, the Bethe result is proportional to the projectile charge squared, Z_1^2 [3,4]. It was thus a surprise when Barkas et al. found that the range of negative pions was longer than that of negative pions of equal momentum[5]. Barkas suggested that the effect was due to a difference in the stopping power stemming from the opposite charge of the charge of the particles[6]. The reduction in the stopping, responsible for the longer range of negative particles as compared to their positively charged antiparticles was later investigated with sigma hyperons[6], pions[7], and muons[8], but these measurements all suffered from the poor quality of the low – velocity particle and antiparticle beams used.

This so – called Barkas effect has been interpreted as a polarization effect in the stopping material depending on the charge of the projectile. It appears as the second term(proportional to Z_1^3) in the implied Born expansion of the energy loss[9].

2. Theory:

2.1 Modified Bethe – Bloch theory:

The basic stopping equation for high velocity particles, as traditionally, was shown as[10]:-

$$S = \frac{kZ_2Z_1^2}{\beta^2}L(\beta) \qquad (1)$$

Where the variable L, called the stopping number, was defined to include the correction factors to the stopping equation for high velocity particles. Traditionally, it is defined as the expansions of the particle's charge[10]

$$L(\beta) = L_0(\beta) + Z_1 L_1(\beta) + Z_1^2 L_2(\beta) \qquad (2)$$

The formalism of Bethe – Bloch theory of stopping power, including various modifications, has been described extensively in several investigations. The stopping power S of an elemental target of atomic number Z_2 and atomic weight A for a projectile of atomic number Z_1 and velocity v ($v = \beta c$, where c is the light velocity), can be expressed by[2]

$$S = \frac{kZ_2 Z_1^2}{\beta^2}\{L_0(\beta) + Z_1 L_1(\beta) + Z_1^2 L_2(\beta)\} \qquad (3)$$

Where $k = 4\pi m_e r_0^2 c^2$ (is a constant), and $r_0 = \frac{e^2}{mc^2}$ (classical electron radius), $L_0(\beta)$ is Born correction $Z_1 L_1(\beta)$ is the Barkas correction or the Z_1^3 correction, and $Z_1^2 L_2(\beta)$ is the Bloch correction or the Z_1^4 correction.

2.2 Barkas correction:

In their derivation of a function for the Barkas effect, based on the harmonic oscillator model, Ashley et al. argued that the effect could be neglected for close collisions (in which the electrons are considered to be free). Thus they introduced a lower limit a_ω of the impact parameter and assumed that the electrons were unbounded for collisions at smaller distances. Using the statistical model, they assumed that a_ω was given approximately by the radius r of the shell of charge associated with the plasma frequency $\omega(r)$, i.e., $a_\omega = \eta r$, where η is of order 1. They derived a function[11]

$$Z_1 L_1 = \frac{\chi Z_1 F_{ARB}(b/x^{1/2})}{Z_1^{1/2} x^{3/2}} \qquad (4)$$

Where $x = \frac{v^2}{Z_2 v_0^2} = \frac{(137\beta)^2}{Z_2}$ **and** $b = \eta\chi Z_2^{1/6}$

The term χ is a free – electron – gas parameter which corrects for binding forces, and has value of about $2^{1/2}$. b was expected to have value between 1 and 2.

Jackson and McCarthy gave a function which can be approximated to better than ±3% by [12]

$$Z_1 L_1 = \frac{B_0 Z_1}{V^2}[g - h\ln(V + \chi^2)]/Z_2^{1/2} \qquad (5)$$

$$V = \frac{v}{v_0 Z_2^{1/2}}$$

Where $1 \leq V \leq 10$, v_0 is the Bohr velocity and for $\chi^2 = 2$, g=0.477, h=0.1385, while for $\chi^2 = 3$, g=0.607, h=0.175. Jackson and McCarthy suggest a different minimum impact parameter $a_\omega = (2m\hbar\omega)^{1/2}$.

Hill and Merzbacher obtained the same result with a quantum – mechanical harmonic oscillator approximation[13].

Lindhard showed that there is a contribution from close collisions which is about the same as that from the distant collisions. His model is too schematic to permit a realistic calculation of the effect[14].

$$Z_1 L_1 = \frac{3\pi Z_1 e^2 \omega}{2mv^3} \ln\left(\frac{v}{1.7\omega a_\omega}\right) \qquad (6)$$

ω is the free electron gas plasma frequency and a_ω is lower limit of the impact parameter for the distant collisions. This high velocity limit is for $v \gg \omega a_\omega$.

The extracted Barkas correction values may be empirically fit using the expression[15].

$$Z_1 L_1 = \frac{L_{low} L_{high}}{L_{low} + L_{high}} \qquad (7)$$

$$L_{low} = 0.001E \quad \text{and} \quad L_{high} = (1.5/E^{0.4}) + 45000/z_2 E^{1.6} \qquad (8)$$

With the energy, E, having units of (KeV/a.m.u.). This expression goes to zero for both low and high values of ion energy. Not that this empirical Barkas correction term is dependent on the other terms in the stopping number, especially the shell correction.

2.3 Barkas correction and density function:

The stopping power of ions moving with velocity smaller than the Fermi velocity $(v < v_f)$ in a homogeneous electron gas is[16]

$$-\frac{dE}{dx} = m\, n\, v\, v_f\, \sigma_{tr}(v_f) \qquad \text{for } v \ll v_f \qquad (9)$$

$$\sigma_{tr}(v_f) = \int_0^{\pi} (1 - \cos\theta)\, \sigma_s(\theta, v_f)\, 2\pi \sin\theta\, d\theta \qquad (10)$$

$$\sigma_s(\theta, v_f) = \frac{Z_1 e^4}{4m^2 v_f^2} \frac{1}{\left[sin^2\frac{\theta}{2} + \left(\frac{\lambda^2}{2a}\right)^2\right]^2} \qquad (11)$$

Where

e, m is the charge and mass of an electron

σ_{tr} is the transport cross section

σ_s is the scattering cross section

a is the atomic radius

n is the electron gas density

θ is the scattering angle

v_f is the Fermi velocity

$\lambda = \frac{\hbar}{m v_f}$ is the Fermi wave length[17]

By substituting eq.(11) into eq.(10), we get

$$\sigma_{tr}(v_f) = 2\pi \int_0^\pi \sin\theta\, d\theta\, (1-\cos\theta) \frac{Z_1 e^4}{4m^2 v_f^2} \left[\frac{1}{\frac{1}{2}(1-\cos\theta) + \left(\frac{\lambda^2}{2a}\right)^2}\right]^2 \qquad (12)$$

By suppose

$$\chi^2 = \frac{v_0}{\pi v_f}$$

$$F = \frac{1}{2}(1+\cos\theta) + \chi^2 \; ; \quad dF = \sin(\theta/2)\, d\theta$$

$$\theta \to 0 \qquad ; \quad F = \chi^2$$

$$\theta \to \pi \qquad ; \quad F = 1 + \chi^2$$

Eq.(12) may be rewritten in the following form

$$\sigma_{tr}(v_f) = \frac{2\pi Z_1 e^4}{m^2 v_f^4} \int_{\chi^2}^{1+\chi^2} dF (F - \chi^2) \frac{1}{F^2} \qquad (13)$$

And by integrating the above equation, we get on

$$\sigma_{tr}(v_f) = \frac{2\pi Z_1 e^4}{m^2 v_f^4} \left[\ln\frac{1+\chi^2}{\chi^2} - \frac{1}{1+\chi^2}\right] \qquad (14)$$

And by taking the second order of the transport cross section in eq.(14), we get on

$$\sigma_{tr}(v_f) = \frac{1}{4}\left(\frac{\pi Z_1 \chi^3 \lambda^2}{1+\chi^2}\right)\left(1 + 4\pi Z_1 \chi^2 \frac{t+\chi^2}{t^{1/2} s} tan^{-1} \frac{t^{1/2}\chi}{s}\right) \qquad (15)$$

Where

$s^2 = t + 4\chi^2 + 4\chi^4$ **and** $t = \sin\theta/2$

By substituting the transport cross section eq.(15) into the stopping power eq.(9) becomes[16]

$$-\frac{dE}{dx} = \frac{4}{3\pi} Z_1^2 C_1(\chi) \frac{v}{v_0} \frac{e^2}{a_0^2} \qquad (16)$$

Where

$\chi^2 = \frac{v_0}{\pi v_f}$ **is the density parameter**

$a_0 = \frac{\hbar^2}{me^2}$ **is the Bohr radius**

$C_1(\chi) = C_1 + C_2$ **is the density function** **(17)**

$$C_1 = \frac{1}{2}\left[\ln\frac{1+\chi^2}{\chi^2} - \frac{1}{1+\chi^2}\right] \qquad (18)$$

$$C_2 = \frac{2\pi Z_1 \chi^3}{3+4\chi^2}\left[4(1+\chi^2)\ln\frac{1/4+\chi^2+\chi^4}{\chi^2+\chi^4} - \ln\frac{1+\chi^2}{\chi^2}\right] \qquad (19)$$

C_1 in eq.(18) is independent on the atomic number of the projectile (Z_1) but it is dependent on the first Born approximation and energy at low velocity. C_2 in eq.(19) is dependent on the atomic number of the projectile (Z_1), therefore it is possible to consider C_2 as a Barkas effect which depends on the velocity of the projectile. The calculation of stopping power for particle at low velocity dependence on the density function as in eq.(17) in which C_1 is independent on Z_1 while C_2 is dependent on Z_1, therefore the density function eq.(17) may be rewritten as

$C_{part}(\chi) = C_1 + C_2$ **for particle** **(20)**

$C_{anti\,part}(\chi) = C_1 + C_2$ **for antiparticle** **(21)**

$\left(-\frac{dE}{dx}\right)_{part} = \alpha(C_1 + C_2)$ **the stopping power for particle** **(22)**

$$\left(-\frac{dE}{dx}\right)_{anti\ part} = \alpha(C_1 - C_2)$$ **the stopping power for antiparticle (23)**

$$\alpha = \frac{4}{3\pi} Z_1^2 \frac{v}{v_0} \frac{e^2}{a_0^2} \quad (24)$$

By division eq.(22) on eq.(23)

$$\frac{(-dE/dx)_{part}}{(-dE/dx)_{anti\ part}} = \frac{\alpha(C_1+C_2)}{\alpha(C_1-C_2)} \quad (25)$$

$$\frac{(-dE/dx)_{part}}{(-dE/dx)_{anti\ part}} = \frac{C_1}{(C_1-C_2)} + \frac{C_2}{(C_1-C_2)} \quad (26)$$

$$\frac{(-dE/dx)_{part}}{(-dE/dx)_{anti\ part}} = \frac{1}{1-C_2/C_1} + \frac{1}{C_1/C_2-1} \quad (27)$$

If,

$$\frac{C}{C_1} = \frac{C_1+C_2}{C_1} = 1 + \frac{C_2}{C_1} \quad (28)$$

When $\frac{C_2}{C_1} \to 0$**, eq.28 becomes** $\frac{C}{C_1} \approx 1$

3. Results and Discussion:

Figure(1) shows the results of stopping power for protons in Al (Z_2=13) and Au (Z_2=79) target as a function of its energy at high velocity which are calculated from the eq.(3) and taking into account all the corrections (Born , Barkas, and Bloch) in the calculations. From the figure, the magnitude of the stopping power decreases with increasing the projectile velocity. At a given value of energy, there is a difference between the stopping power in Al and Au target and the stopping power appears to be increasing with decreasing the atomic number of the target (Z_2), therefore the stopping in Al target is larger than that in Au target at low velocity but at high velocity, the difference becomes small and the stopping powers in both target are approaching because the stopping power of ions shows a non monotonic dependence on the atomic number of the target (Z_2) at low velocity.

Figure(2) shows the results of Barkas correction for protons in Al (Z_2=13) and Au (Z_2=79) target as a function of its energy at high velocity which are calculated from the eq.(7). From the figure, the magnitude of the Barkas decreases with increasing the projectile velocity. At a given value of energy, there is a difference between the Barkas correction in Al and Au

target and the Barkas correction appears to be increasing with decreasing the atomic number of the target (Z_2), therefore the Barkas correction in Al target is larger than that in Au target at low velocity but at high velocity, the difference becomes small and Barkas correction in both target is approaching because the effect of Barkas correction becomes insignificant.

Figure(3) shows the percent contribution of Barkas correction to stopping number as a function of energy for protons in Al (Z_2=13) and Au (Z_2=79) which are calculated from eq.(7) at high velocity. From the figure, the percent of Barkas correction decreases with increasing the energy and becomes small at high velocity and the Barkas correction for Al target contributes less than 1% for all energies above (10MeV) while for Au target it contributes less than 1% for all energies (15MeV), therefore there is a divergence in values at low velocities and convergence in values of percent contribution of Barkas correction at high velocities because it becomes significant at low velocity and it decreases with increasing the energy. The percent Barkas correction in Au target is larger than that in Al target because the targets are different and for each one a specific atomic number and there are a number of corrections to the stopping number for each target. In Al target, the other corrections to the stopping number are larger than that in Au target, therefore the percent contribution of Barkas correction in Au to the stopping power is larger than that in Al. In general, the corrections are more important at low velocity and they appear to be decreasing with increasing the energy, therefore at high velocity, the values of corrections are approaching and become very small.

Figure(4)shows the results of stopping power (a) for particle (Z_1=2) and antiparticle (Z_1=-2) and (b) for particle (Z_1=5) and antiparticle (Z_1=-5) as a function of density parameter (χ) which are calculated from eq.(16). From the figure, the stopping power of both particles increases with decreasing the density parameter (χ) since this mean increasing energy. The stopping power depends on the atomic number of the projectile (Z_1) and it increases with increasing atomic number of it (Z_1), therefore the stopping power of particle is larger than that of antiparticle. At high values of density parameter (χ) (low velocity), there is a difference in stopping between particle and antiparticle, therefore we can distinguish between them because this belongs to the effect of Barkas correction, but at low value of density parameter (χ) (high velocity), the stopping powers of particle and antiparticle are approaching and the difference becomes small because the Barkas correction decreases with increasing the velocity and becomes insignificant at high velocity.

Figure(5) shows the results of proportion of stopping power (a) of particle (Z_1=2) to antiparticle (Z_1=-2) and (b) of particle (Z_1=5) to antiparticle (Z_1=-5) as a function of density parameter (χ) which are calculated from eq.(25). From the figure, when the density parameter (χ) increases (velocity decreases), the difference in stopping power between particle and antiparticle will also increase because the effect of Barkas correction appears clearly at low velocity.

Figure(6) shows the results of proportion of density function to the first Born approximation (C/C_1) for (a) particle (Z_1=2) and antiparticle (Z_1=-2) and for (b) particle (Z_1=5) and antiparticle (Z_1=-5) as a function of density parameter (χ) which are calculated from eq.(28). From the figure, when the density parameter (χ) is very small, $\left(\frac{C}{C_1} \to 1\right)$ and the proportion of stopping power of particle to antiparticle becomes nearly one, therefore we can not distinguish between particle and antiparticle but when the density parameter (χ) is large, $\frac{C}{C_1}$ is given by the eq.(28). C_2 is dependent on the atomic number of the target (Z_2), therefore $\frac{C}{C_1}$ changes in opposite direction related to particle and antiparticle.

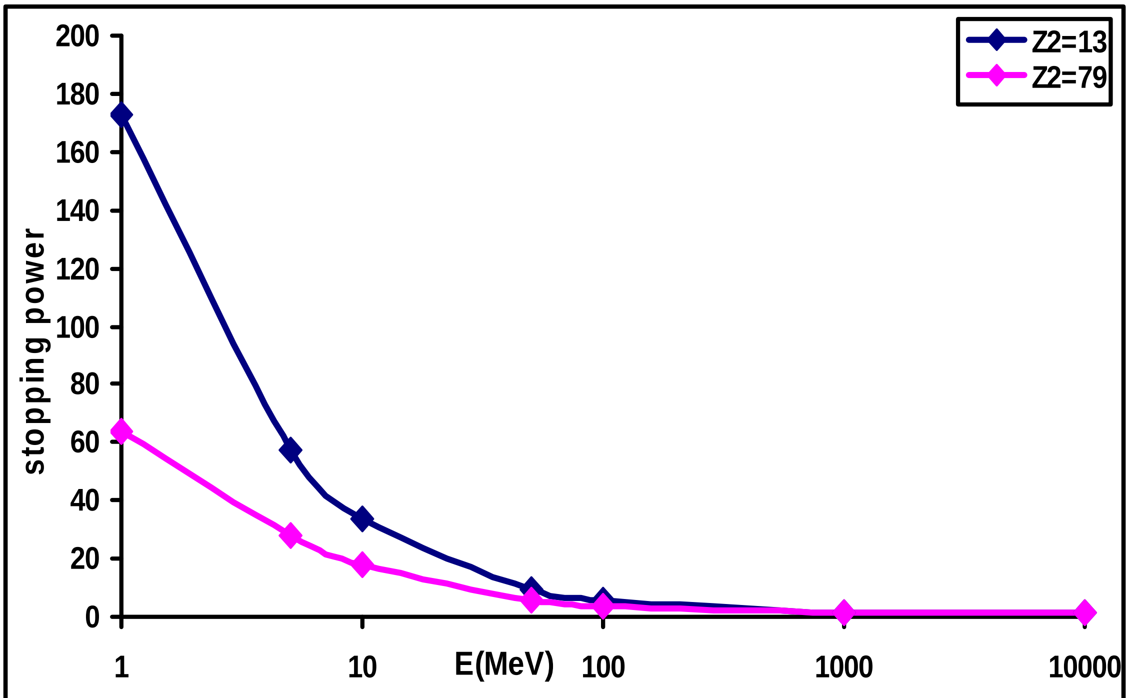


Fig.(1) Stopping power of proton in Al(Z2=13) and Au(Z2=79)

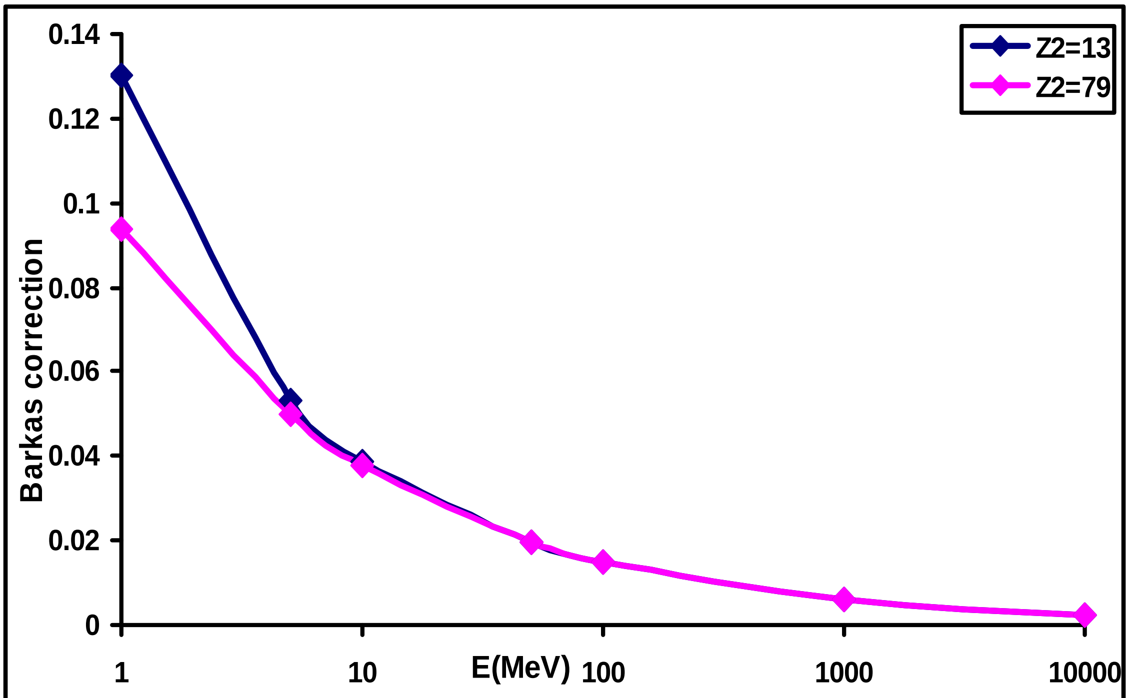


Fig.(2) Barkas correction for proton in Al(Z2=13) and Au(Z2=79)

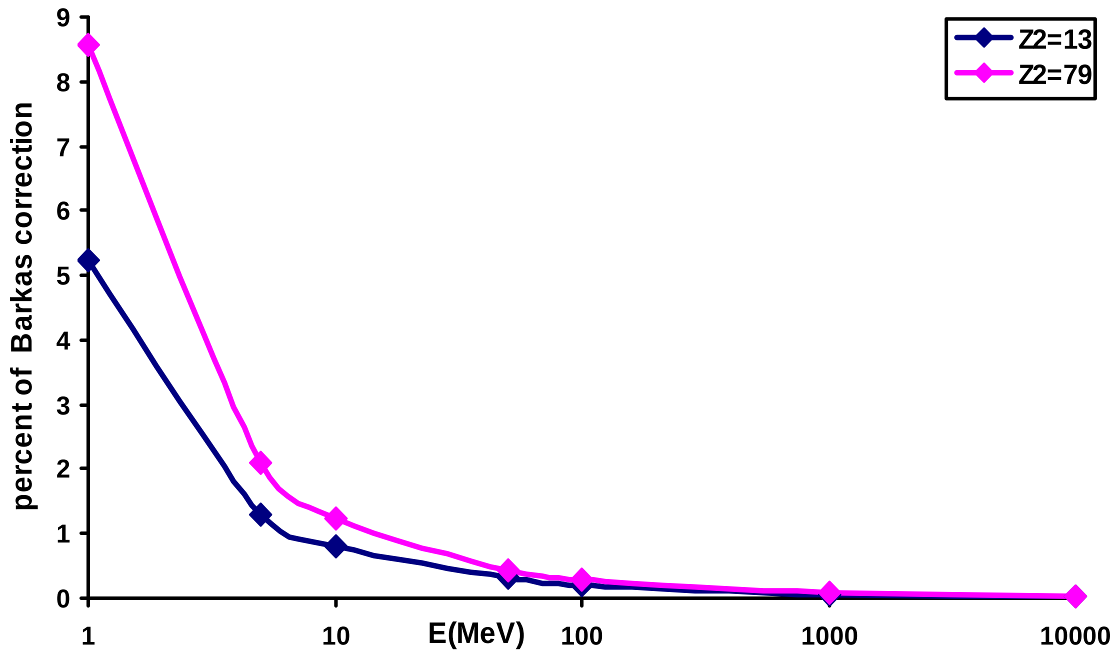
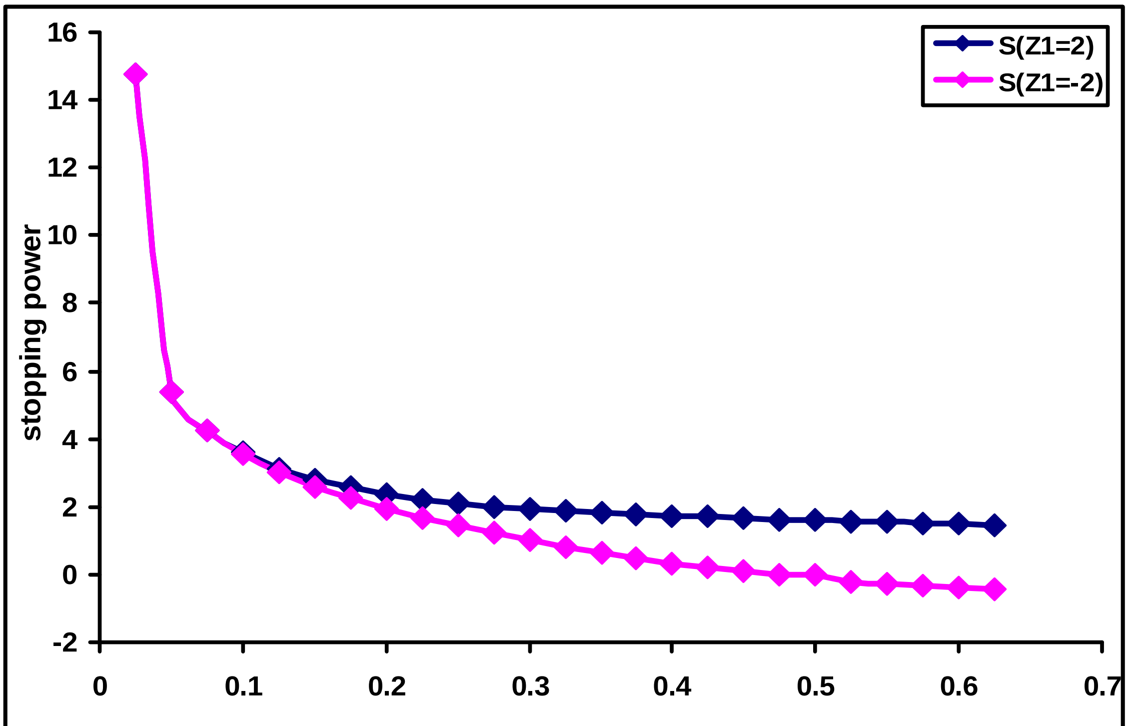
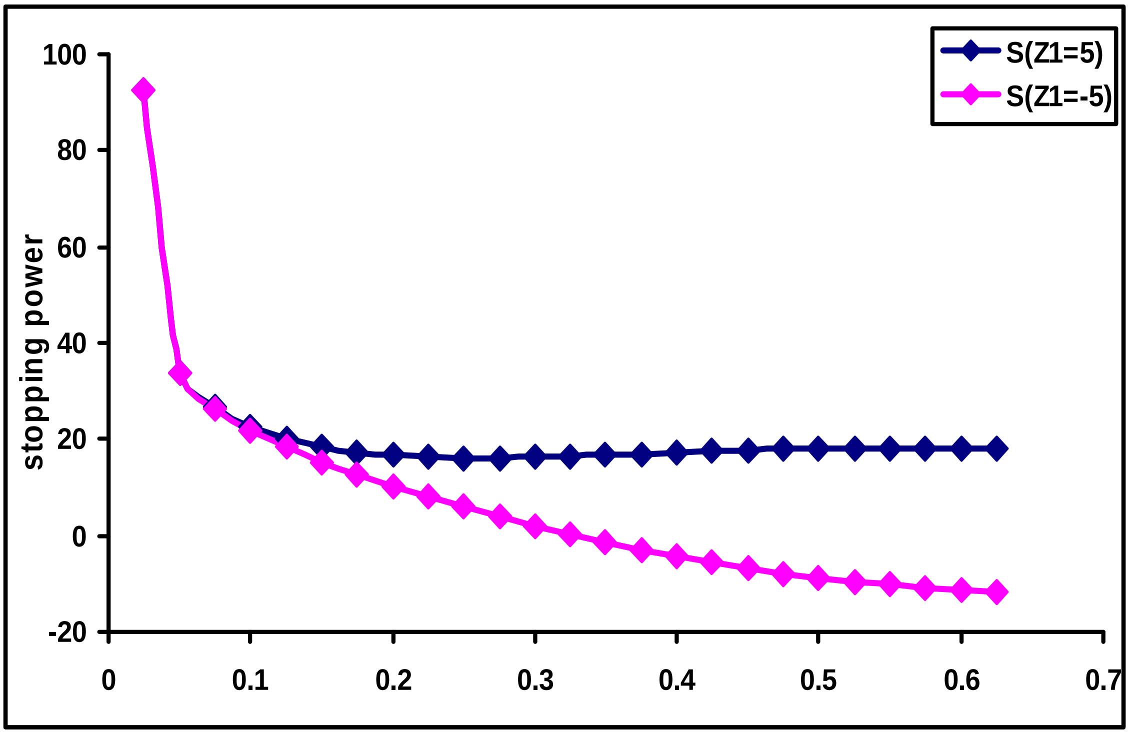


Fig.(3) Percent contribution of Barkas correction to stopping number for proton in Al(Z2=13) and Au(Z2=79)



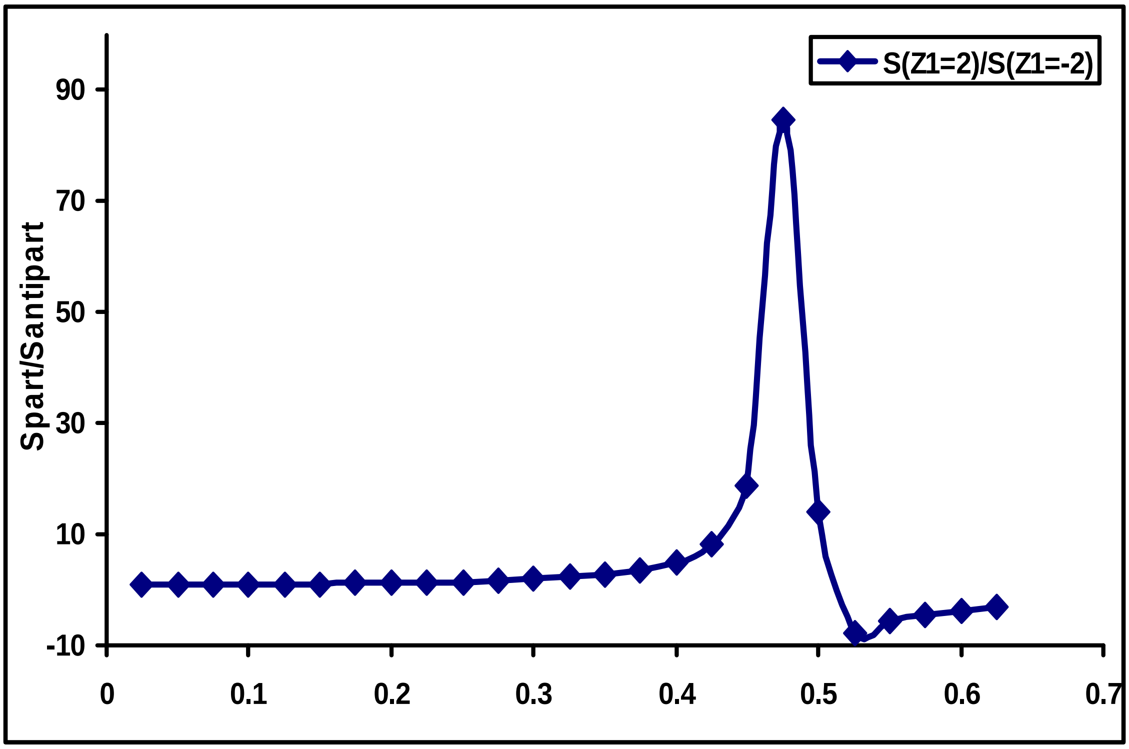
χ

Fig.(4-a)Stopping power of particle ($Z_1 = 2$) and antiparticle ($Z_1 = -2$)with density parameter (χ)



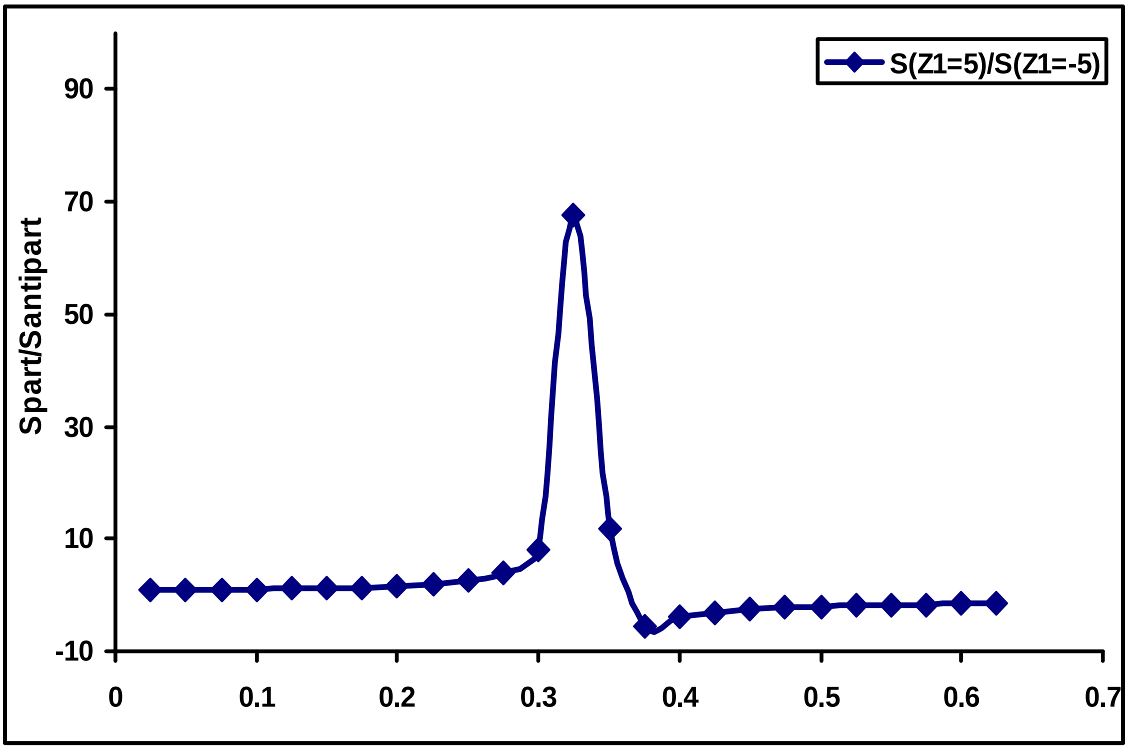
χ

Fig.(4-b)Stopping power of particle ($Z_1 = 5$) and antiparticle ($Z_1 = -5$) with density parameter (χ)



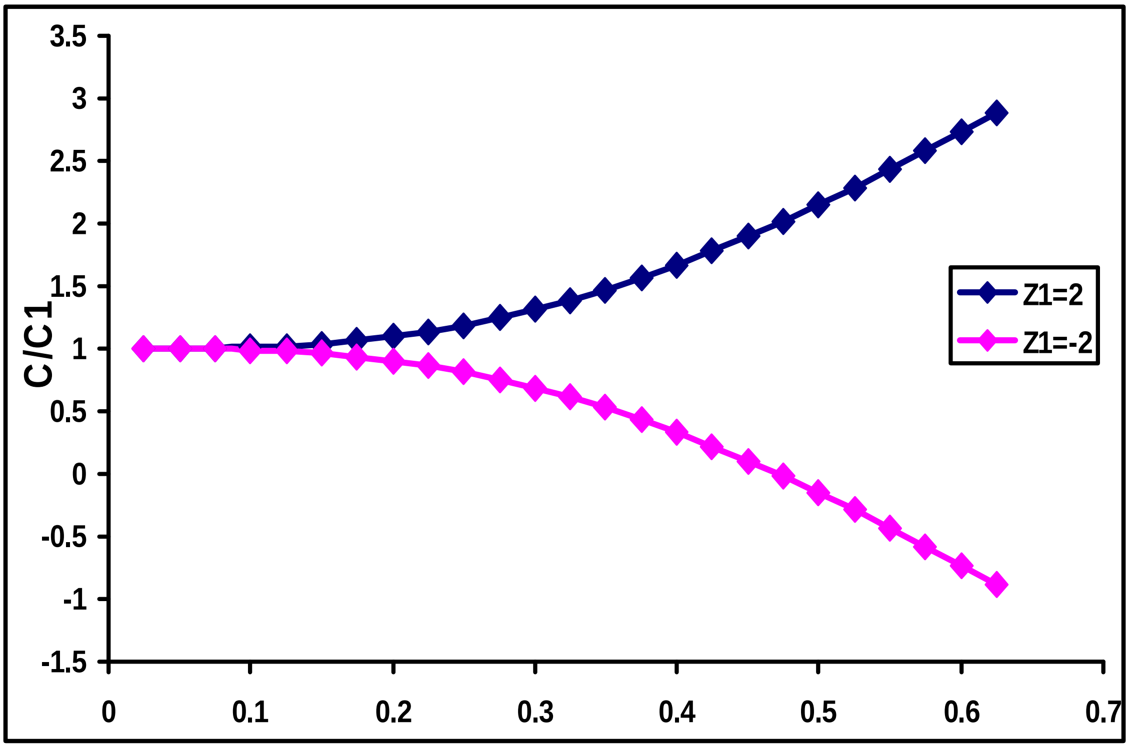
χ

Fig.(5-a) Proportion of stopping power of particle ($Z_1 = 2$) to antiparticle ($Z_1 = -2$) with density parameter (χ)



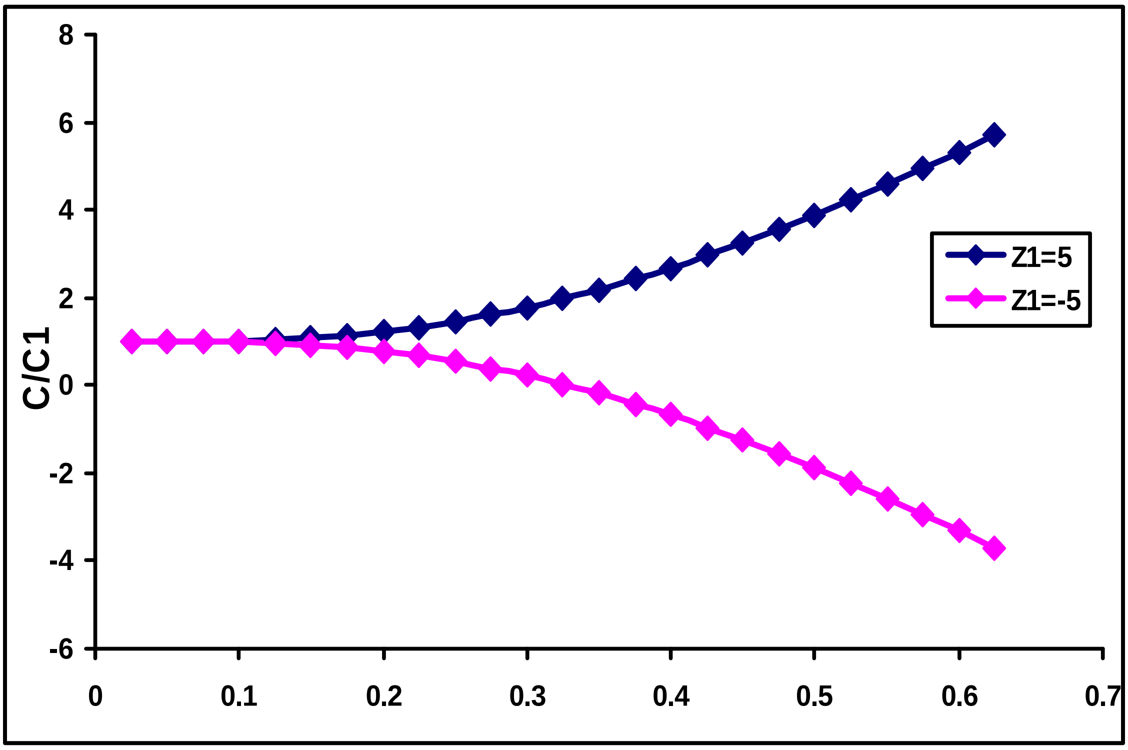
χ

Fig.(5-b) Proportion of stopping power of particle ($Z_1 = 5$) to antiparticle ($Z_1 = -5$) with density parameter (χ)



χ

Fig.(6-a) Proportion of density function to the first Born approximation (C/C_1) of particle ($Z_1 = 2$) and antiparticle ($Z_1 = -2$) with density parameter (χ)



χ

Fig.(6-b) Proportion of density function to the first Born approximation (C/C_1) of particle ($Z_1 = 5$) and antiparticle ($Z_1 = -5$) with density parameter (χ)

4. Conclusions:

The phenomena of ionization in ion – atom collisions is closely related to the associated energy loss by the colliding ion. According to Bethe, the stopping power of a point fast charged particle penetrating through matter is proportional to Z_1^2, the square of its charge, which is based on the first Born approximation. When the projectile velocity(v) decreases, there is a deviation from simple first – order perturbation theory stopping predictions (higher – order Z_1 effect). In addition to the Z_1^2 dependence for stopping power, terms with odd powers (Z_1^3) in Z_1 lead to a different stopping behavior of positively and negatively charged particles (particle and antiparticle), this difference is called the Barkas correction which is interpreted as being due to polarization of the target material.

Barkas correction depends on the projectile velocity and it decreases with increasing velocity of the projectile, and reaches a maximum when this is comparable to the velocity of the electrons in the medium. At high velocity the Barkas effect becomes insignificant because the ion will be moving too fast to cause initial motion of target electrons. Barkas correction

also depends on the projectile and target atomic number (Z_1) and (Z_2). At a given projectile velocity the magnitude of Barkas correction increases with increasing Z_1 and decreasing Z_2.

Moreover, Barkas correction is dependent on the sign of the projectile. However, a positive charge will pull these target towards it as it approaches, increasing the local electron density and increasing its energy loss relative to that of a negatively charged particle, while a negative charge will repel target electrons, decreasing local electron density. For the positive projectile charge, Z_1L_1 will be positive, hence the Barkas correction will contribute to the stopping (increasing its magnitude) while it will be subtracted from the negative projectile stopping (decreasing its magnitude) because of the negative value of Z_1L_1. Hence Barkas effect may be extracted directly by assuming it was proportional to exact one – half the difference between particle and antiparticle (positive and negative) stopping power in the same target, at the same velocity. The Barkas factor was determined by dividing this stopping difference by the Bethe – Bloch prefactor.

The stopping power of particle is larger than that of antiparticle at low velocity because of the effect of Barkas correction therefore we can distinguish between particle and antiparticle at low velocity through the density function which consists of two terms C_1 and C_2. C_1 is independent on the atomic number of projectile (Z_1) while C_2 is dependent on (Z_1) therefore, C_1 represents the Barkas correction which decreases with increasing the projectile energy and its effect becomes very small at high energy.

References:

[1] Alexandru Csete, " Experimental Investigations of the Energy Loss of Slow Protons and Antiprotons in Matter ", institute of physics and Astronomy University of Aarhus(2002).

[2] H. Ammi, R. Zemih, S. Mammeri, and M. Allab, Nucl. Inst. and Meth., In physics Research, B, Vol.230, pp.68 – 72 (2005).

[3] H. Bethe, Ann. Physic ,Vol.5, pp.325(1930).

[4] U. Fano, Ann. Rev. Nucl. Sci. Vol.13(1963).

[5] W. H. Barkas, W. Birnbaun, and F. M. Smith, Phys. Rev.,Vol.101, pp.778(1956).

[6] W. H. Barkas, N. J. Dyer, and H. H. Heckman, Phys. Rev. Lett.,Vol.11, pp.26 (1963).

[7] H. H. Heckman and P. J. Lindstrom, Phys. Rev., Lett., Vol.22, pp.871(1969).

[8] W. Wilhelm, H. Daniel, and F. J. Hartmann, Phys. Lett., B, Vol.98, pp.33(1981).

[9] L. H. Andersen et al., phys. Rev. Lett. Vol.62, N.15 pp.1731 – 1734(1989).

[10] J. F. Ziegler, J. Appl. phys. Rev. Appl. phys. , Vol.85, pp.1249 – 1272 (1999).

[11]J.C Ashley, R. H. Ritchie, and W. Brandt, phys. Rev. , Vol.5, N.7, pp.2393(1972).

[12] J. D. Jackson and R. L. McCarthy ,phys. Rev., B,Vol.6, N.11, pp.4131(1972).

[13] K.W. Hill and E. Merzbacher, phys. Rev., A, Vol. 9, pp.156 (1974).

[14] J. Lindhard, Nucl. Inst. Methods, Vol.132, pp.1(1976).

[15] J. F. Ziegler, J. Appl. phys. Rev. Appl. phys., Vol.85, pp.1249 – 1272 (1999).

[16] A.H. Sorenson, Nucl. Inst. and Meth. In physics Research, B, Vol.48, pp.10 – 13 (1990).

[17] E. Bonderup, Lecture Nots on the penetration of charged particles through matter, (University of Aarhus), Denmark up published (1978).